Calcul $t(\theta)$

(I)

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

$$v = \frac{2\pi}{T} a \; X(\theta) \quad \text{avec} \quad X(\theta) = \left(\frac{1+2e\cos\theta+e^2}{1-e^2}\right)^{1/2} = \frac{dl}{dt}$$

$$dl^2 = dx^2 + dy^2 = (r^2 + r'^2)\, d\theta^2$$

$$dl = (r^2 + r'^2)^{1/2} d\theta = \left(r^2 + r^2 \frac{e^2\sin^2\theta}{(1+e\cos\theta)^2}\right)^{1/2} d\theta$$

$$dl = r\left(1 + \frac{e^2\sin^2\theta}{(1+e\cos\theta)^2}\right)^{1/2} = r\,\frac{(1+e^2\cos^2\theta+2e\cos\theta+e^2\sin^2\theta)^{1/2}}{(1+e\cos\theta)}\, d\theta$$

$$dl = r\,\frac{(1+2e\cos\theta+e^2)^{1/2}}{(1+e\cos\theta)}\, d\theta$$

$$dt = \frac{dl}{v} = r\,\frac{\cancel{(1+2e\cos\theta+e^2)^{1/2}}}{(1+e\cos\theta)} \cdot \frac{1}{\frac{2\pi}{T} a \frac{\cancel{(1+2e\cos\theta+e^2)^{1/2}}}{(1-e^2)^{1/2}}}\, d\theta$$

$$dt = \frac{T}{2\pi}\cdot\frac{r}{a}\cdot\frac{(1-e^2)^{1/2}}{(1+e\cos\theta)} = \frac{T}{2\pi}\,\frac{\cancel{a}(1-e^2)}{(1+e\cos\theta)}\,\frac{1}{\cancel{a}}\cdot\frac{(1-e^2)^{1/2}}{(1+e\cos\theta)}\, d\theta$$

$$dt = \left(\frac{T}{2\pi}\right)\frac{(1-e^2)^{3/2}}{(1+e\cos\theta)^2}\, d\theta$$

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pour $e \ll 1$ $\quad dt = \left(\frac{T}{2\pi}\right)(1+e\cos\theta)^{-2}\, d\theta \;\#\; \left(\frac{T}{2\pi}\right)(1-2e\cos\theta)\, d\theta$

$$t = \int_0^\theta dt = \frac{T}{2\pi}\left[\theta - 2e\sin\theta\right] \qquad \begin{cases} \theta = 0 & t = 0 \\ \theta = \pi & t = \frac{T}{2} \quad (\text{ok}) \end{cases}$$

$$t = \left(\frac{T}{2\pi}\right)(1-e^2)^{3/2}\int_0^\theta \frac{1}{(1+e\cos\theta)^2}\, d\theta = \left(\frac{T}{2\pi}\right)(1-e^2)^{3/2}\cdot \dot{I}$$