VII. *On the Means of discovering the Distance, Magnitude, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendish, *Esq. F. R. S. and A. S.*

Read November 27, 1783.

DEAR SIR, Thornhill, May 26, 1783.

THE method, which I mentioned to you when I was last in London, by which it might perhaps be possible to find the distance, magnitude, and weight of some of the fixed stars, by means of the diminution of the velocity of their light, occurred to me soon after I wrote what is mentioned by Dr. PRIESTLEY in his History of Optics, concerning the diminution of the velocity of light in consequence of the attraction of the sun; but the extreme difficulty, and perhaps impossibility, of procuring the other data necessary for this purpose appeared to me to be such objections against the scheme, when I first thought of it, that I gave it then no farther consideration. As some late observations, however, begin to give us a little more chance of procuring some at least of these data, I thought it would not be amiss, that astronomers should be apprized of the method, I propose (which, as far as I know,

has not been ſuggeſted by any one elſe) leſt, for want of being aware of the uſe, which may be made of them, they ſhould neglect to make the proper obſervations, when in their power; I ſhall therefore beg the favour of you to preſent the following paper on this ſubject to the Royal Society.

I am, &c.

THE very great number of ſtars that have been diſcovered to be double, triple, &c. particularly by Mr. HERSCHEL *, if we apply the doctrine of chances, as I have heretofore done in my "Enquiry into the probable Parallax, &c. of the Fixed "Stars," publiſhed in the Philoſophical Tranſactions for the year 1767, cannot leave a doubt with any one, who is properly aware of the force of thoſe arguments, that by far the greateſt part, if not all of them, are ſyſtems of ſtars ſo near to each other, as probably to be liable to be affected ſenſibly by their mutual gravitation; and it is therefore not unlikely, that the periods of the revolutions of ſome of theſe about their principals (the ſmaller ones being, upon this hypotheſis, to be conſidered as ſatellites to the others) may ſome time or other be diſcovered.

2. Now the apparent diameter of any central body, round which any other body revolves, together with their apparent diſtance from each other, and the periodical time of the revolv-

* See his Catalogue of Stars of this kind, publiſhed in the Philoſophical Tranſactions for the year 1782, which is indeed a moſt valuable preſent to the aſtronomical world. By a happy application of very high magnifyiug powers to his teleſcopes, and by a moſt perſevering induſtry in obſerving, he has made a very wonderful progreſs in this branch of aſtronomy, in which almoſt nothing of any conſequence had been done by any one before him.

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ing body being given, the density of the central body will be given likewise. See Sir ISAAC NEWTON's Prin. b. III. pr. VIII. cor. 1.

3. But the density of any central body being given, and the velocity any other body would acquire by falling towards it from an infinite height, or, which is the same thing, the velocity of a comet revolving in a parabolic orbit, at its surface, being given, the quantity of matter, and consequently the real magnitude of the central body, would be given likewise.

4. Let us now suppose the particles of light to be attracted in the same manner as all other bodies with which we are acquainted; that is, by forces bearing the same proportion to their *vis inertiæ*, of which there can be no reasonable doubt, gravitation being, as far as we know, or have any reason to believe, an universal law of nature. Upon this supposition then, if any one of the fixed stars, whose density was known by the above-mentioned means, should be large enough sensibly to affect the velocity of the light issuing from it, we should have the means of knowing its real magnitude, &c.

5. It has been demonstrated by Sir ISAAC NEWTON, in the 39th proposition of the first book of his Principia, that if a right line be drawn, in the direction of which a body is urged by any forces whatsoever, and there be erected at right angles to that line perpendiculars every where proportional to the forces at the points, at which they are erected respectively, the velocity acquired by a body beginning to move from rest, in consequence of being so urged, will always be proportional to the square root of the area described by the aforesaid perpendiculars. And hence,

6. If such a body, instead of beginning to move from rest, had already some velocity in the direction of the same line,

when

when it began to be urged by the aforeſaid forces, its velocity would then be always proportional to the ſquare root of the ſum or difference of the aforeſaid area, and another area, whoſe ſquare root would be proportional to the velocity which the body had before it began to be ſo urged; that is, to the ſquare root of the ſum of thoſe areas, if the motion acquired was in the ſame direction as the former motion, and the ſquare root of the difference, if it was in a contrary direction. See cor. 2. to the aboveſaid propoſition.

7. In order to find, by the foregoing propoſition, the velocity which a body would acquire by falling towards any other central body, according to the common law of gravity, let C in the figure (tab. III.) repreſent the centre of the central body, towards which the falling body is urged, and let CA be a line drawn from the point C, extending infinitely towards A. If then the line RD be ſuppoſed to repreſent the force, by which the falling body would be urged at any point D, the velocity which it would have acquired by falling from an infinite height to the place D would be the ſame as that which it would acquire by falling from D to C with the force RD, the area of the infinitely extended hyperbolic ſpace ADRB, where RD is always inverſely proportional to the ſquare of DC, being equal to the rectangle RC contained between the lines RD and CD. From hence we may draw the following corollaries.

8. Cor. 1. The central body DEF remaining the ſame, and conſequently the forces at the ſame diſtances remaining the ſame likewiſe, the areas of the rectangles RC, *r*C will always be inverſely as the diſtances of the points D, *d* from C, their ſides RD, *rd* being inverſely in the duplicate ratio of the ſides CD, C*d*: and therefore, becauſe the velocity of a body falling from an infinite height towards the point C, is always in the ſub-

sub-duplicate ratio of these rectangles, it will be in the sub-duplicate ratio of the lines CD, C*d* inversely. Accordingly the velocities of comets revolving in parabolic orbits are always in the sub-duplicate ratio of their distances from the sun inversely; and the velocities of the planets, at their mean distances (being always in a given ratio to the velocity of such comets, *viz.* in the sub-duplicate ratio of 1 to 2) must necessarily observe the same law likewise.

9. Cor. 2. The magnitude of the central body remaining the same, the velocity of a body falling towards it from an infinite height will always be, at the same distance from the point C, taken any where without the central body, in the sub-duplicate ratio of its density; for in this case the distance C*d* will remain the same, the line *rd* only being increased or diminished in the proportion of the density, and the rectangle *r*C consequently increased or diminished in the same proportion.

10. Cor. 3. The density of the central body remaining the same, the velocity of a body falling towards it from an infinite height will always be as its semi-diameter, when it arrives at the same proportional distance from the point C; for the weights, at the surfaces of different spheres of the same density are as their respective semi-diameters; and therefore the sides RD and CD, or any other sides *rd* and C*d*, which are in a given ratio to those semi-diameters, being both increased or diminished in the same proportion, the rectangles RC or *r*C will be increased or diminished in the duplicate ratio of the semi-diameter CD, and consequently the velocity in the simple ratio of CD.

11. Cor. 4. If the velocity of a body falling from an infinite height towards different central bodies is the same, when it arrives at their surfaces, the density of those central bodies must be

in the duplicate ratio of their ſemi-diameters inverſely; for by the laſt cor. the denſity of the central body remaining the ſame, the rectangle RC will be in the duplicate ratio of CD; in order therefore that the rectangle RC may always remain the ſame, the line RD muſt be inverſely, as CD, and conſequently the denſity inverſely, as the ſquare of CD.

12. Cor. 5. Hence the quantity of matter contained in thoſe bodies muſt be in the ſimple ratio of their ſemi-diameters directly; for the quantity of matter being always in a ratio compounded of the ſimple ratio of the denſity, and the triplicate ratio of their ſemi-diameters, if the denſity is in the inverſe duplicate ratio of the ſemi-diameters, this will become the direct triplicate and inverſe duplicate, that is, when the two are compounded together, the ſimple ratio of the ſemi-diameters,

13. The velocity a body would acquire by falling from an infinite height towards the ſun, when it arrived at his ſurface, being, as has been ſaid before in article 3d, the ſame with that of a comet revolving in a parabolic orbit in the ſame place, would be about 20,72 times greater than that of the earth in its orbit at its mean diſtance from the ſun; for the mean diſtance of the earth from the ſun, being about 214,64 of the ſun's ſemidiameters, the velocity of ſuch a comet would be greater at that diſtance than at the diſtance of the earth from the ſun, in the ſub-duplicate ratio of 214,64 to 1, and the velocity of the comet being likewiſe greater than that of planets, at their mean diſtances, in the ſub-duplicate ratio of 2 to 1; theſe, when taken together, will make the ſub-duplicate ratio of 429,28 to 1, and the ſquare root of 429,28 is 20,72, very nearly.

14. The

14. The same result would have been obtained by taking the line RD proportional to the force of gravity at the sun's surface, and DC equal to his semi-diameter, and from thence computing a velocity, which should be proportional to the square root of the area RC when compared with the square root of another area, one of whose sides should be proportional to the force of gravity at the surface of the earth; and the other should be, for instance, equal to 16 feet, 1 inch, the space a body would fall through in one second of time, in which case it would acquire a velocity of 32 feet, 2 inches per second. The velocity thus found compared with the velocity of the earth in its orbit, when computed from the same elements, necessarily gives the same result. I have made use of this latter method of computation upon a former occasion, as may be seen in Dr. PRIESTLEY's History of Optics, p. 787, &c. but I have rather chosen to take the velocity from that of a comet, in the article above, on account of its greater simplicity, and its more immediate connexion with the subject of this paper.

15. The velocity of light, exceeding that of the earth in its orbit, when at its mean distance from the sun, in the proportion of about 10.310 to 1, if we divide 10.310 by 20,72, the quotient 497, in round numbers, will express the number of times, which the velocity of light exceeds the velocity a body could acquire by falling from an infinite height towards the sun, when it arrived at his surface; and an area whose square root should exceed the square root of the area RC, where RD is supposed to represent the force of gravity at the surface of the sun, and CD is equal to his semi-diameter, in the same proportion, must consequently exceed the area RC in the proportion of 247.009, the square of 497 to 1.

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

17. But if the ſemi-diameter of a ſphære, of the ſame denſity with the ſun, was of any other ſize leſs than 497 times that of the ſun, though the velocity of the light emitted from ſuch a body, would never be wholly deſtroyed, yet would it always ſuffer ſome diminution, more or leſs, according to the magnitude of the ſaid ſphære; and the quantity of this diminution may be eaſily found in the following manner: Suppoſe S to repreſent the ſemi-diameter of the ſun, and aS to repreſent the ſemi-diameter of the propoſed ſphære; then, as appears from what has been ſhewn before, the ſquare root of the difference between the ſquare of 497 S and the ſquare of aS will be always proportional to the ultimately remaining velocity, after it has ſuffered all the diminution, it can poſſibly ſuffer from this cauſe; and conſequently the difference between the whole velocity of light, and the remaining velocity, as found above, will be the diminution of its velocity. And hence the diminution of the velocity of light emitted from the ſun, on account of it's gravitation towards that body, will be ſomewhat leſs than a 494.000dth part of the velocity which it would have had if no ſuch diminution had taken place; for the ſquare of 497 being 247.009, and the ſquare of 1 being 1, the diminution of the velocity will be the difference between

the square root of 247.009, and the square root of 247.008, which amounts, as above, to somewhat less than one 494.000th part of the whole quantity.

18. The same effects would likewise take place, according to article 11, if the semi-diameters were different from those mentioned in the two last articles, provided the density was greater or less in the duplicate ratio of those semi-diameters inversely.

19. The better to illustrate this matter, it may not be amiss to take a particular example. Let us suppose then, that it should appear from observations made upon some one of those double stars above alluded to, that one of the two performed its revolution round the other in 64 years, and that the central one was of the same density with the sun, which it must be, if its apparent diameter, when seen from the other body, was the same as the apparent diameter of the sun would be if seen from a planet revolving round him in the same period: let us further suppose, that the velocity of the light of the central body was found to be less than that of the sun, or other stars whose magnitude was not sufficient to affect it sensibly, in the proportion of 19 to 20. In this case then, according to article 17, the square root of 247.009 SS must be to the square root of the difference between 247.009 SS and aaSS as 20 to 19. But the squares of 20 and 19 being 400 and 361, the quantity 247.009 SS must therefore be to the difference between this quantity and aaSS in the same proportion, that is as 247.009 to 222.925,62; and aaSS must consequently be equal to 24.083, 38 SS, whose square root 155,2 S nearly, or, in round numbers, 155 times the diameter of the sun, will be the diameter of the central star sought.

20. As the squares of the periodical times of bodies, revolving round a central body, are always proportional to the cubes of their mean distances, the distance of the two bodies from each other must therefore, upon the foregoing suppositions, be sixteen times greater in proportion to the diameter of the central body, than the distance of the earth from the sun in proportion to his diameter; and that diameter being already found to be also greater than that of the sun in the proportion of 155,2 to 1, this distance will consequently be greater than that of the earth and sun from each other in the proportion of 16 times 155,2, that is 2483,2 to 1.

21. Let us farther suppose, that from the observations, the greatest distance of the two stars in question appeared to be only one second; we must then multiply the number 2483,2 by 206.264,8, the number of seconds in the radius of a circle, and the product 512.196.750 will shew the number of times which such a star's distance from us must exceed that of the sun. The quantity of matter contained in such a star would be $\frac{}{155,2}^{3}$ or 3.738.308 times as much as that contained in the sun; its light, supposing the sun's light to take up 8′. 7″. in coming to the earth, would, with its common velocity, require 7.900 years to arrive at us, and 395 years more on account of the diminution of that velocity; and supposing such a star to be equally luminous with the sun, it would still be very sufficiently visible, I apprehend, to the naked eye, notwithstanding its immense distance.

22. In the elements which I have employed in the above computations, I have supposed the diameter of the central star to have been observed, in order to ascertain its density, which cannot be known without it; but the diameter of such a star is

much too small to be observed by any telescopes yet existing, or any that it is probably in the power of human abilities to make; for the apparent diameter of the central star, if of the same density with the sun, when seen from another body, which would revolve round it in 64 years, would be only the 1717th part of the distance of those bodies from each other, as will appear from multiplying 107,32, the number of times the sun's diameter is contained in his distance from the earth, by 16, the greater proportional distance of the revolving body, corresponding to 64 years instead of 1. Now the 1717th part of a second must be magnified 309.060 times in order to give it an apparent diameter of three minutes; and three minutes, if the telescopes were mathematically perfect, and there was no want of distinctness in the air, would be but a very small matter to judge of*.

23. But

* In Mr. HERSCHEL's Observations upon the Fixed Stars abovementioned, almost all of them are represented as appearing with a well-defined round disc. That this is not the real disc, but only an optical appearance, occasioned perhaps by the constitution of the eye, when the pencil, by which objects are seen, is so exceedingly small as those which he employed upon this occasion, is very manifest, from the observations themselves, of which indeed Mr. HERSCHEL seems to be himself sufficiently aware: if it were not so, the intensity of the light of these stars must either be exceedingly inferior indeed to that of the sun, or they must be immensely larger, otherwise they must have a very sensible parallax; for the sun, if removed to 10.000.000 times his present distance, would still, I apprehend, be of about the brightness of the stars of the sixth magnitude; in which case he must be magnified 1.000.000 times to make his apparent disc of any sensible magnitude; or, on the other hand, if he was only removed to a thousandth part of that distance, then he must be less luminous in the proportion of 1.000.000 to 1, to make him appear no brighter than a star of the sixth magnitude. Now the sun's diameter being contained nearly 215 times in the diameter of the earth's orbit, the annual parallax therefore of such a body in that case, if it was placed in the pole of the ecliptic,

23. But though there is not the least probability that this element, so essential to be known, in order to determine with precision the exact distance and magnitude of a star, can ever be obtained, where it is in the same circumstances, or nearly the same, with those above supposed, yet the other elements, such as perhaps may be obtained, are sufficient to determine the distance, &c. with a good deal of probability, within some moderate limits; for in whatever ratio the real distance of the two stars may be greater or less than the distance supposed, the density of the central star must be greater or less in the sixth power of that ratio inversely; for the periodic time of the revolving body being given, the quantity of matter contained in the central body must be as the cube of their distance from each other. See Sir I. NEWTON's Prin. b. 3d. pr. 8th. cor 3d. But the quantity of matter in different bodies, at whose surfaces the velocity acquired by falling from an infinite height is the same, must be, according to art. 12, directly as their semi-diameters; the semi-diameters therefore of such bodies must be in the triplicate ratio of the distance of the revolving body; and consequently their densities, by art. 11, being in the inverse duplicate ratio of their semi-diameters, must be in the inverse sextuplicate ratio of the distance of the revolving body. Hence if the real distance should be greater or less than that supposed, in the proportion of two or three to one, the density of the central body must be less or greater, in the first case, in the proportion of 64, or in the latter of 729 to 1.

ecliptic, would be 215 times its apparent diameter; and as the bright star in Lyrâ appeared to Mr. HERSCHEL about a third part of a second in diameter, if this was its real disc, and it was no bigger than the sun, it would consequently have an annual parallax in the pole of the ecliptic of about 72″.

24. There

24. There is also another circumstance, from which perhaps some little additional probability might be derived, with regard to the real distance of a star, such as that we have supposed; but upon which however, it must be acknowledged, that no great stress can be laid, unless we had some better analogy to go upon than we have at present. The circumstance I mean is the greater specific brightness which such a star must have, in proportion as the real distance is less than that supposed, and *vice versâ*; since, in order that the star may appear equally luminous, its specific brightness must be as the fourth power of its distance inversely; for the diameter of the central star being as the cube of the distance between that and the revolving star, and their distance from the earth being in the simple ratio of their distance from each other, the apparent diameter of the central star must be as the square of its real distance from the earth, and consequently, the surface of a sphære being as the square of its diameter, the area of the apparent disc of such a star must be as the fourth power of its distance from the earth; but in whatever ratio the apparent disc of the star is greater or less, in the same ratio inversely must be the intensity of its light, in order to make it appear equally luminous. Hence, if its real distance should be greater or less than that supposed in the proportion of 2 or 3 to 1, the intensity of its light must be less or greater, in the first case, in the proportion of 16, or, in the latter of 81 to 1.

25. According to Mons. Bouguer (see his Traité d'Optique) the brightness of the sun exceeds that of a wax candle in no less a proportion than that of 8000 to 1. If therefore the brightness of any of the fixed stars should not exceed that of our common candles, which, as being something less luminous than wax,

wax, we will ſuppoſe in round numbers to be only one 10.000dth part as bright as the ſun, ſuch a ſtar would not be viſible at more than an 100dth part of the diſtance, at which it would be viſible, if it was as bright as the ſun. Now becauſe the ſun would ſtill appear, I apprehend, as luminous, as the ſtar Sirius, when removed to 400.000 times his preſent diſtance, ſuch a body, if no brighter than our common candles, would only appear equally luminous with that ſtar at 4000 times the diſtance of the ſun, and we might then begin to be able, with the beſt teleſcopes, to diſtinguiſh ſome ſenſible apparent diameter of it; but the apparent diameters of the ſtars of the leſs magnitudes would ſtill be too ſmall to be diſtinguiſhable even with our beſt teleſcopes, unleſs they were yet a good deal leſs luminous, which may poſſibly however be the caſe with ſome of them; for, though we have indeed very ſlight grounds to go upon with regard to the ſpecific brightneſs of the fixed ſtars compared with that of the ſun at preſent, and can therefore only form very uncertain and random conjectures concerning it, yet from the infinite variety which we find in the works of the creation, it is not unreaſonable to ſuſpect, that very poſſibly ſome of the fixed ſtars may have ſo little natural brightneſs in proportion to their magnitude, as to admit of their diameters having ſome ſenſible apparent ſize, when they ſhall come to be more carefully examined, and with larger and better teleſcopes than have been hitherto in common uſe.

26. With regard to the ſun, we know that his whole ſurface is extremely luminous, a very ſmall and temporary interruption ſometimes from a few ſpots only excepted. This univerſal and exceſſive brightneſs of the whole ſurface is probably owing to an atmoſphære, which being luminous throughout, and

and in some measure also transparent, the light, proceeding from a considerable depth of it, all arrives at the eye; in the same manner as the light of a great number of candles would do, if they were placed one behind another, and their flames were sufficiently transparent to permit the light of the more distant ones to pass through those that were nearer, without any interruption.

27. How far the same constitution may take place in the fixed stars we don't know; probably however it may do so in many; but there are some appearances with regard to a few of them, which seem to make it probable, that it does not do so universally. Now, if I am right in supposing the light of the sun to proceed from a luminous atmosphære, which must necessarily diffuse itself equally over the whole surface, and I think there can be very little doubt that this is really the case, this constitution cannot well take place in those stars, which are in some degree periodically more and less luminous, such as that in Collo Ceti, &c. It is also not very improbable, that there is some difference from that of the sun, in the constitution of those stars, which have sometimes appeared and sometimes disappeared, of which that in the constellation of Cassiopeia is a notable instance. And if those conjectures are well founded which have been formed by some philosophers concerning stars of these kinds, that they are not wholly luminous, or at least not constantly so, but that all, or by far the greatest part of their surfaces is subject to considerable changes, sometimes becoming luminous, and at other times being extinguished; it is amongst the stars of this sort, that we are most likely to meet with instances of a sensible apparent diameter, their light being much more likely not to be so great in proportion as that of the sun, which, if removed to four hundred thousand times

his present distance would still appear, I apprehend, as bright as Sirius, as I have observed above; whereas it is hardly to be expected, with any telescopes whatsoever, that we should ever be able to distinguish a well defined disc of any body of the same size with the sun at much more than ten thousand times his distance.

28. Hence the greatest distance at which it would be possible to distinguish any sensible apparent diameter of a body as dense as the sun cannot well greatly exceed five hundred times ten thousand, that is, five million times the distance of the sun; for if the diameter of such a body was not less than five hundred times that of the sun, its light, as has been shewn above, in art. 16. could never arrive at us.

29. If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us; or if there should exist any other bodies of a somewhat smaller size, which are not naturally luminous; of the existence of bodies under either of these circumstances, we could have no information from sight; yet, if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis; but as the consequences of such a supposition are very obvious, and the consideration of them somewhat beside my present purpose, I shall not prosecute them any farther.

30. The diminution of the velocity of light, in case it should be found to take place in any of the fixed stars, is the principal phænomenon whence it is proposed to discover their distance, &c. Now the means by which we may find what this diminution amounts to, seems to be supplied by the difference which would be occasioned in consequence of it, in the refrangibility of the light, whose velocity should be so diminished. For let us suppose with Sir ISAAC NEWTON (see his Optics, prop. VI. paragr. 4 and 5) that the refraction of light is occasioned by a certain force impelling it towards the refracting medium, an hypothesis which perfectly accounts for all the appearances. Upon this hypothesis the velocity of light in any medium, in whatever direction it falls upon it, will always bear a given ratio to the velocity it had before it fell upon it, and the sines of incidence and refraction will, in consequence of this, bear the same ratio to each other with these velocities inversely. Thus, according to this hypothesis, if the sines of the angles of incidence and refraction, when light passes out of air into glass, are in the ratio of 31 to 20, the velocity of light in the glass must be to its velocity in air in the same proportion of 31 to 20. But because the areas, representing the forces generating these velocities, are as the squares of the velocities, see art. 5. and 6. these areas must be to each other as 961 to 400. And if 400 represents the area which corresponds to the force producing the original velocity of light, 561, the difference between 961 and 400, must represent the area corresponding to the additional force, by which the light was accelerated at the surface of the glass.

31. In art. 19. we supposed, by way of example, the velocity of the light of some particular star to be diminished in the

ratio of 19 to 20, and it was there obſerved, that the area repreſenting the remaining force which would be neceſſary to generate the velocity 19, was therefore properly repreſented by $\frac{361}{400}$dth parts of the area, that ſhould repreſent the force that would be neceſſary to generate the whole velocity of light, when undiminiſhed. If then we add 561, the area repreſenting the force by which the light is accelerated at the ſurface of the glaſs, to 361, the area repreſenting the force which would have generated the diminiſhed velocity of the ſtar's light, the ſquare root of 922, their ſum, will repreſent the velocity of the light with the diminiſhed velocity, after it has entered the glaſs. And the ſquare root of 922 being 30,364, the ſines of incidence and refraction of ſuch light out of air into glaſs will conſequently be as 30,364 to 19, or what is equal to it, as 31,96 to 20 inſtead of 31 to 20, the ratio of the ſines of incidence and refraction, when the light enters the glaſs with its velocity undiminiſhed.

32. From hence a priſm, with a ſmall refracting angle, might perhaps be found to be no very inconvenient inſtrument for this purpoſe: for by ſuch a priſm, whoſe refracting angle was of one minute, for inſtance, the light with its velocity undiminiſhed would be turned out of its way 33″, and with the diminiſhed velocity 35″, 88 nearly, the difference between which being almoſt 2″. 53‴, would be the quantity by which the light, whoſe velocity was diminiſhed, would be turned out of its way more than that whoſe velocity was undiminiſhed.

33. Let us now be ſuppoſed to make uſe of ſuch a priſm to look at two ſtars, under the ſame circumſtances as the two ſtars in the example above-mentioned, the central one of which ſhould be large enough to diminiſh the velocity of its light one twentieth part, whilſt the velocity of the light of the other,

which

which was supposed to revolve about it as a satellite, for want of sufficient magnitude in the body from whence it was emitted, should suffer no sensible diminution at all. Placing then the line, in which the two faces of the prism would intersect each other, at right angles to a line joining the two stars; if the thinner part of the prism lay towards the same point of the heavens with the central star, whose light would be most turned out of its way, the apparent distance of the stars would be increased $2''.53'''$ and consequently become $3''.53'''$ instead of $1''.$ only, the apparent distance supposed above in art. 21. On the contrary, if the prism should be turned half way round, and its thinner part lye towards the same point of the heavens with the revolving star, their distance must be diminished by a like quantity, and the central star therefore would appear $1''.53'''$ distant from the other on the opposite side of it, having been removed from its place near three times the whole distance between them.

34. As a prism might be made use of for this purpose, which should have a much larger refracting angle than that we have proposed, especially if it was constructed in the achromatic way, according to Mr. DOLLOND's principles, not only such a diminution, as one part in twenty, might be made still more distinguishable; but we might probably be able to discover considerably less diminutions in the velocity of light, as perhaps a hundredth, a two-hundredth, a five-hundredth, or even a thousandth part of the whole, which, according to what has been said above, would be occasioned by sphæres, whose diameters should be to that of the sun, provided they were of the same density, in the several proportions nearly of 70, 50, 30, and 22 to 1 respectively.

35. If such a diminution of the velocity of light, as that above supposed, should be found really to take place, in consequence

quence of its gravitation towards the bodies from whence it is emitted, and there should be several of the fixed stars large enough to make it sufficiently sensible, a set of observations upon this subject might probably give us some considerable information with regard to many circumstances of that part of the universe, which is visible to us. The quantity of matter contained in many of the fixed stars might from hence be judged of, with a great degree of probability, within some moderate limits; for though the exact quantity must still depend upon their density, yet we must suppose the density most enormously different from that of the sun, and more so, indeed, than one can easily conceive to take place in fact, to make the error of the supposed quantity of matter very wide of the truth, since the density, as has been shewn above in art. 11. and 12. which is necessary to produce the same diminution in the velocity of light, emitted from different bodies, is as the square of the quantity of matter contained in those bodies inversely.

36. But though we might possibly from hence form some reasonable guess at the quantity of matter contained in several of the fixed stars; yet, if they have no luminous satellites revolving about them, we shall still be at a loss to form any probable judgment of their distance, unless we had some analogy to go upon for their specific brightness, or had some other means of discovering it; there is, however, a case that may possibly occur, which may tend to throw some light upon this matter.

37. I have shewn in my Enquiry into the probable Parallax, &c. of the Fixed Stars, published in the Philosophical Transactions for the year 1767, the extremely great probability there is, that many of the fixed stars are collected together into groups; and that the Pleiades in particular constitute one of these

these groups. Now of the stars which we there see collected together, it is highly probable, as I have observed in that paper, that there is not one in a hundred which does not belong to the group itself; and by far the greatest part, therefore, according to the same idea, must lye within a sphære, a great circle of which is of the same size with a circle, which appears to us to include the whole group. If we suppose, therefore, this circle to be about 2°. in diameter, and consequently only about a thirtieth part of the distance at which it is seen, we may conclude, with the highest degree of probability, that by far the greatest part of these stars do not differ in their distances from the sun by more than about one part in thirty, and from thence deduce a sort of scale of the proportion of the light which is produced by different stars of the same group or system in the Pleiades at least; and, by a somewhat probable analogy, we may do the same in other systems likewise. But having yet no means of knowing their real distance, or specific brightness, when compared either with the sun or with one another, we shall still want something more to form a farther judgment from.

38. If, however, it should be found, that amongst the Pleiades, or any other like system, there are some stars that are double, triple, &c. of which one is a larger central body, with one or more satellites revolving about it, and the central body should likewise be found to diminish the velocity of its light; and more especially, if there should be several such instances met with in the same system; we should then begin to have a kind of measure both of the distance of such a system of stars from the earth, and of their mutual distances from each other. And if several instances of this kind should occur in different groups or systems of stars, we might also, perhaps, begin to form

form some probable conjectures concerning the specific density and brightness of the stars themselves, especially if there should be found any general analogy between the quantity of the diminution of the light and the distance of the system deduced from it; as, for instance, if those stars, which had the greatest effect in diminishing the velocity of light should in general give a greater distance to the system, when supposed to be of the same density with the sun, we might then naturally conclude from thence, that they are less in bulk, and of greater specific density, than those stars which diminish the velocity of light less, and *vice versâ*. In like manner, if the larger stars were to give us in general a greater or less quantity of light in proportion to their bulk, this would give us a kind of analogy, from whence we might perhaps form some judgment of the specific brightness of the stars in general; but, at all adventures, we should have a pretty tolerable measure of the comparative brightness of the sun and those stars, upon which such observations should be made, if the result of them should turn out agreeable to the ideas above explained.

39. Though it is not improbable, that a few years may inform us, that some of the great number of double, triple stars, &c. which have been observed by Mr. HERSCHEL, are systems of bodies revolving about each other, especially if a few more observers, equally ingenious and industrious with himself could be found to second his labours; yet the very great distance at which it is not unlikely many of the secondary stars may be placed from their principals, and the consequently very long periods of their revolutions *, leave very little room to hope that

* If the sun, when removed to 10.000.000 times his present distance, would still appear as bright as a star of the fifth magnitude, which I apprehend to be pretty

that any very great progress can be made in this subject for many years, or perhaps some ages to come; the above outlines, therefore, of the use that may be made of the observations upon the double stars, &c. provided the particles of light should be subject to the same law of gravitation with other bodies, as in all probability they are, and provided also that some of the stars should be large enough sensibly to diminish their velocity, will, I hope, be an inducement to those, who may have it in their power, to make these observations for the benefit of future generations at least, how little advantage soever we may expect from them ourselves; and yet very possibly some observations of this sort, and such as may be made in a few years, may not only be sufficient to do something, even at present, but also to shew, that much more may be done hereafter, when these observations shall become more numerous, and have been continued for a longer period of years.

pretty near the truth, any satellite revolving round such a star, provided the star was not either of less specific brightness, or of greater density than the sun, must, if it appeared at its greatest elongation, at the distance of one second only from its principal, be between three and four hundred years in performing one revolution; and the time of the revolution of the very small star near α Lyræ, if it is a satellite to this latter, and its principal is of the same specific brightness and density with the sun, could hardly be less than eight hundred years, though 37″ the distance at which it is placed from it, according to Mr. Herschel's observations, should happen to be its greatest distance. These periodical times, however, are computed from the above distances, upon the supposition of the star, that revolves as a satellite, being very much smaller than the central one, so as not to disturb its place sensibly; for if the two stars should contain equal, or nearly equal, quantities of matter, the periodical times might be somewhat less, on account of their revolving about their common centre of gravity, in circles of little more than half as great a diameter as that in which the satellite must revolve upon the other supposition.