

Montrons que $MA^2 + MB^2 = 2MI^2 + AB^2/2$

On a $\overrightarrow{AI} + \overrightarrow{BI} = 0$ car I milieu de AB

Et $MA^2 + MB^2 = (\overrightarrow{MI} + \overrightarrow{IA})^2 + (\overrightarrow{MI} + \overrightarrow{IB})^2$

$$= MI^2 + 2\overrightarrow{MI} \cdot \overrightarrow{IA} + IA^2 + MI^2 + 2\overrightarrow{MI} \cdot \overrightarrow{IB} + IB^2$$

$$= 2MI^2 + 2\overrightarrow{MI}(\overrightarrow{IA} + \overrightarrow{IB}) + 2IA^2 \text{ car } IA = IB$$

$$= 2MI^2 + AB^2/2 \text{ car } \overrightarrow{IA} + \overrightarrow{IB} = 0 \text{ et } IA = AB/2$$