EXO Math

1a)

$$y_G = \frac{M y_{G1} + m y_{G2}}{M + m}$$

on pose $c = \frac{M}{\rho S} \Rightarrow \boxed{M = c\rho S}$

$$m = \frac{S\cancel{H}\rho}{\cancel{H}} h$$

$$m = S\rho h$$

$$S = \frac{m}{\rho h}$$

$$\Rightarrow c = \frac{M}{\rho m/\rho h}$$

$$c = \frac{M}{m} \cdot h$$

$$\boxed{m = \frac{M}{c} h}$$

$$y_G = \frac{c\rho S^{H}/_{2} + \frac{M}{c} \cdot h^2/_2}{c\rho S + \frac{M}{c} h}$$

$$y_G = \frac{1}{2} \frac{c\rho SH + \frac{M}{c} \cdot h^2}{c\rho S + M/c \cdot h}$$

$$y_G = \frac{1}{2} \frac{c^2 \rho SH + Mh^2}{c^2 \rho S + Mh}$$

$$y_G = \frac{c^2\rho SH + c\rho S h^2}{c^2 \rho S + c\rho S h} \frac{1}{2}$$

$$\boxed{y_G = \frac{1}{2} \frac{cH + h^2}{c + h}}$$

1b) étude variation :

$$y_G' = \frac{U'V - V'U}{V^2}$$

$$y_G' = \frac{2h(2c + 2h) - 2(cH + h^2)}{(2c + 2h)^2}$$

$$y_G' = \frac{4hc + 4h^2 - 2cH - 2h^2}{(2c + 2h)^2}$$

$$y_G' = \frac{2h^2 + 4hc - 2cH}{(2c + 2h)^2}$$

» Polynôme degré 2 ⇒ Parabole

$a > 0$ ⇒ Parabole avec comme extremum un minimum

$(2c+2h)^2 > 0$

$\Rightarrow$ étude de $2h^2 + 4hc - 2CH = 0$

$\Delta = 16C^2 + 16CH$

$$h_1 = \frac{-4C + \sqrt{16C^2 + 16CH}}{2}$$

$h_2 < 0$ impossible par raport à l'exo (hauteur d'eau)

1c) Calcul valeur minimale de Y_G :

$$Y_G = \frac{CH + \left(\frac{-4C + \sqrt{16C^2+16CH}}{2}\right)^2}{C + \left(\frac{-4C + \sqrt{16C^2+16CH}}{2}\right)}$$

$$Y_G = \frac{CH + 4C^2 - 2C\sqrt{16C^2+16CH} + 4C^2 + 4CH}{2C - 4C + \sqrt{16C^2+16CH}}$$

$$Y_G = \frac{C\left(5H + 8C - 2\sqrt{16C^2+16CH}\right)}{-2C + \sqrt{16C^2+16CH}}$$??