V1 = (2,0,1,1) , v2 = (- 2, 2, 4, 0) , v3 = (-1, 1 ,2, -6) , v4 = (1, 1, 0, 0)

Cherchons une base orthonormale de B par le procédé de Gram-Schmidt

v1 ' = v1 = (2, 0, 1, 1) donc u1 = $\frac{v1'}{\|v1'\|}$ = $\frac{(2,0,1,1)}{\sqrt{6}}$

v2' = v2 – proj $_{u1}$ v2 = (-2, 2, 4, 0) - $\frac{(-2,2,4,0).(2,0,1,1)\sqrt{6}^2}{(2,0,1,1).(2,0,1,1)\sqrt{6}^2}$ (2,0,1,1)

= (-2, 2, 4, 0) - $\frac{0}{6}$ (2,0,1,1) = (-2, 2, 4, 0)

donc u2 = $\frac{v2'}{\|v2'\|}$ = $\frac{(-2,2,4,0)}{\sqrt{24}}$

v3' = v3 - proj $_{u2}$ v3 - proj $_{u2}$ v3

= (-1,1,2,-6) - $\frac{(-1,1,2,-6).(2,0,1,1)}{(2,0,1,1).(2,0,1,1)}$ (2,0,1,1) - $\frac{(-1,1,2,-6).(-2,2,4,0)}{(-2,2,4,0).(-2,2,4,0)}$ (-2,2,4,0)

= (-1,1,2,-6) - $\frac{6}{6}$ (2,0,1,1) - $\frac{12}{24}$ (-2,2,4,0) = (2, 0, 1, -5)

donc u3 = $\frac{v3'}{\|v3'\|}$ = $\frac{(2,0,1,-5)}{\sqrt{30}}$

v4' = v4 -proj $_{u1}$ v4 - proj $_{u2}$ v4 - proj $_{u3}$ v4

$$= (1,1,0,0)-\frac{(1,1,0,0).(2,0,1,1)}{(2,0,1,1).(2,0,1,1)}(2,0,1,1)-\frac{(1,1,0,0).(-2,2,4,0)}{(-2,2,4,0).(-2,2,4,0)}(-2,2,4,0)-\frac{(1,1,0,0).(2,0,1,-5)}{(2,0,1,-5).(2,0,1,-5)}(2,0,1,-5)$$

$$= (1,1,0,0)-\frac{2}{6}(2,0,1,1)-\frac{0}{24}(-2,2,4,0)-\frac{2}{30}(2,0,1,-5)$$

$$= (1,1,0,0)-(\frac{2}{3},0,\frac{1}{3},\frac{1}{3})-(\frac{2}{15},0,\frac{1}{15},\frac{-1}{3}) = (\frac{3}{15},1,\frac{-6}{15},0) = \frac{1}{15}*(3,15,-6,0)$$

$$\text{donc u4} = \frac{v4'}{\|v4'\|} = \frac{\frac{(3,15,-6,0)}{15}}{\frac{\sqrt{3^2+15^2+6^2}}{\sqrt{15^2}}} = \frac{(3,15,-6,0)}{15}*\frac{15}{\sqrt{3^2+15^2+6^2}}$$

$$= \frac{(3,15,-6,0)}{\sqrt{270}}$$