

$$x^2 y' + y = x$$

$$\hat{f}(x) = x^\lambda \sum_{m \geq 0} a_m x^m ; \quad \lambda \in \mathbb{R}, \quad a_m \in \mathbb{R}$$

on a :

$$\hat{f}'(x) = \left[x^\lambda \cdot (a_0 + a_1 x + a_2 x^2 + \dots) \right]'$$

$$\hat{f}'(x) = \lambda x^{\lambda-1} \cdot (a_0 + a_1 x + a_2 x^2 + \dots) + x^\lambda \cdot (0 + a_1 + 2a_2 x + 3a_3 x^2 + \dots)$$

$$= \lambda x^{\lambda-1} \cdot \sum_{m \geq 0} a_m x^m + x^\lambda \sum_{m \geq 1} m a_m x^{m-1}$$

$$= \lambda x^{\lambda-1} \cdot \sum_{m \geq 0} a_m x^m + x^{\lambda-1} \sum_{m \geq 1} m a_m x^m$$

$$x^2 \cdot \hat{f}'(x) = \lambda x^{\lambda+1} \cdot \sum_{m \geq 0} a_m x^m + x^{\lambda+1} \sum_{m \geq 1} m a_m x^m$$

$$x^2 \cdot \hat{f}'(x) + \hat{f}(x) = \lambda x^{\lambda+1} \cdot \sum_{m \geq 0} a_m x^m + x^{\lambda+1} \sum_{m \geq 1} m a_m x^m + x^\lambda \sum_{m \geq 0} a_m x^m$$