$$= \sum_{n\geq 0} a_n x^n \times \left(\lambda x^{\lambda+1} + x^\lambda\right) + x^{\lambda+1} \sum_{n\geq 0} n a_n x^n$$

$$= \lambda \sum_{n\geq 0} a_n x^{n+\lambda+1} + \sum_{n\geq 0} a_n x^{n+\lambda} + \sum_{n\geq 0} n a_n x^{n+\lambda+1} =$$

la premiere somme les $a_n = 0$ sauf

a_n tel que $n+\lambda+1 = 1 \iff n = -\lambda$

d'où $\lambda \sum_{n\geq 0} a_n x^{n+\lambda+1} = \lambda a_{-\lambda} x$

la $2^{ème}$ somme les $a_n = 0$ sauf

a_n tel que $n+\lambda = 1 \iff n = 1-\lambda$

d'où $\sum_{n\geq 0} a_n x^{n+\lambda} = a_{1-\lambda} \cdot x$

la $3^{ème}$ somme les $a_n = 0$ sauf

a_n tel que $n+\lambda+1 = 1 \iff n = -\lambda$

d'où $\sum_{n\geq 0} n a_n x^{n+\lambda+1} = -\lambda a_{-\lambda} \cdot x$

Alors : $\lambda a_{-\lambda} \cdot x + a_{1-\lambda} \cdot x - \lambda a_{-\lambda} \cdot x = x$

Donc : $a_{1-\lambda} = 1$; $\forall n \neq 1-\lambda$; $a_n = 0$