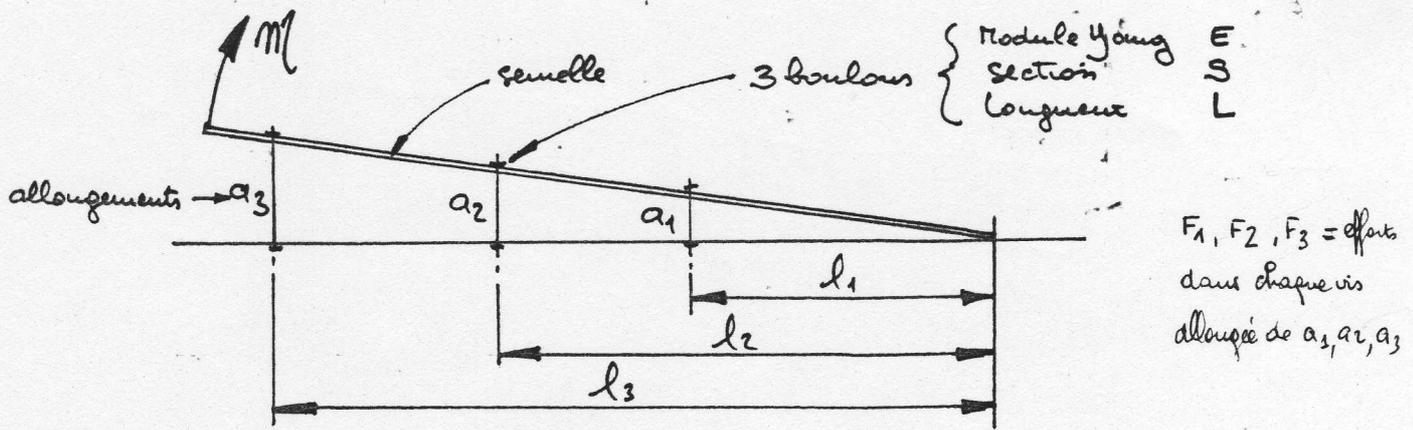# EXO 3: ALLONGEMENT - SYSTEME HYPERSTATIQUE

Allongements :
$$a_1 = \frac{F_1 L_1}{E_1 S_1} \qquad a_2 = \frac{F_2 L_2}{E_2 S_2} \qquad a_3 = \frac{F_3 L_3}{E_3 S_3}$$

Couples :
$$F_1 l_1 + F_2 l_2 + F_3 l_3 = M$$

angle :
$$\frac{a_1}{l_1} = \frac{a_2}{l_2} = \frac{a_3}{l_3}$$

$$\frac{a_1 E_1 S_1}{L_1} . l_1 + \frac{a_2 E_2 S_2}{L_2} . l_2 + \frac{a_3 E_3 S_3}{L_3} . l_3 = M$$

$$\frac{a_1 E_1 S_1}{L_1} . l_1 + \frac{a_2 l_2}{l_1} . \frac{E_2 S_2 l_2}{L_2} + \frac{a_1 l_3}{l_1} . \frac{E_3 S_3 l_3}{L_3} = M$$

$$\frac{a_1}{l_1}\left(\frac{E_1 S_1 l_1^2}{L_1} + \frac{E_2 S_2 l_2^2}{L_2} + \frac{E_3 S_3 l_3^2}{L_3} \right) = M$$

$$\boxed{a_1 = \frac{M . l_1}{\left(\frac{E_1 S_1 l_1^2}{L_1} + \frac{E_2 S_2 l_2^2}{L_2} + \frac{E_3 S_3 l_3^2}{L_3} \right)}}$$

Cas particulier si
$$\left|\begin{array}{l} E_1 = E_2 = E_3 = E \\ L_1 = L_2 = L_3 = L \\ S_1 = S_2 = S_3 = S \end{array}\right.$$

$$\boxed{a_1 = \frac{M . l_1 . L}{E . S (l_1^2 + l_2^2 + l_3^2)}}$$

Exemple : si $l_1 = 100 \quad l_2 = 200 \quad l_3 = 300\,mm \qquad S = 10\,mm^2$
$$M = 50\,000\,daN.mm \qquad L = 60\,mm \qquad E = 20000\,daN.mm^{-2}$$

$$a_1 = \frac{50000 . 100 . 60}{20000 . 10 (100^2 + 200^2 + 300^2)} = 0,0107143\,mm$$

$$a_2 = \frac{a_1 l_2}{l_1} = 0,0214228\,mm \qquad a_3 = \frac{a_1 l_3}{l_1} = 0,0321429\,mm$$

$$F_1 = \frac{a_1}{L} . E.S = 35,71... \, daN$$

$$F_2 = \frac{a_2}{L} . E.S = 71,42... \, daN$$

$$F_3 = \frac{a_3}{L} . E.S = 107,14... \, daN$$

Vérif : $35,71 . 100 + 71,42 . 200 + 107,14 . 300$
$$= 50000 \, daN.mm$$