

power of c (c^β). In the case where $q \neq 0$, the force would also depend on q ; this dependence must be such that the gravitational mass of a box that contains moving elastic material points depends on the velocity of motion of the points in the same way as the gravitational mass. In view of the results of the old theory of relativity, this can only be achieved by the postulate

$$\mathfrak{R}_s = \frac{-m \text{ grad } c \cdot c^\beta}{\sqrt{1 - \frac{q^2}{c^2}}} \cdot \text{const.}$$

If one substitutes \mathfrak{R}_{xs} in the equations of motion accordingly, then one can prove that $\mathfrak{R}_{xa}\dot{x} + \mathfrak{R}_{ya}\dot{y} + \mathfrak{R}_{za}\dot{z}$ can represent a time derivative only in the case where the constants α and β are given values that result in the equations of motion presented in the previous paper. One will therefore have to stick with them and to the expression (4) for the force that results from them if one does not want to give up the whole theory (determination of the static gravitational field by c). [25]

Thus, it seems that the only way to avoid a contradiction with the reaction principle is to replace equations (3) and (3a) with other equations homogeneous in c for which the reaction principle is satisfied when the force postulate (4) is applied. I hesitate to take this step because by doing so I am leaving the territory of the unconditional equivalence principle. It seems that the latter can be maintained for infinitely small fields only. Our derivations of the equations of motion of the material point and of the electromagnetic equations do not thereby become illusory, because they apply equations (2) only to infinitesimally small spaces. For example, these derivations can also be tied to the more general equations [26]

$$\xi = x + \frac{c \frac{dc}{dx}}{2} t^2,$$

$$\eta = y,$$

$$\zeta = z,$$

$$\tau = ct,$$

where c is an arbitrary function of x .—

If one reformulates the integral

$$\int \frac{\Delta c}{c} \text{ grad } c d\tau$$

(which is extended over an arbitrary volume) in a suitable manner, one can easily see for oneself that the reaction principle is satisfied if one retains (4) while replacing equation (3a) with the equation

[27] (3b)
$$c \Delta c - \frac{1}{2} (\text{grad } c)^2 = kc^2 \sigma,$$

which can also be brought into the form

(3b')
$$\Delta (\sqrt{c}) = \frac{k}{2} \sqrt{c} \sigma,$$

where σ denotes the density of the ponderable matter, or the density of the ponderable matter multiplied by the energy density as measured by pocket instruments. From these equations it follows that

[28] (5)
$$\left\{ \begin{array}{l} \mathfrak{F}_x = -\sigma \frac{\partial c}{\partial x} = \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} \quad \text{etc.,} \\ \text{where} \\ ckX_x = \frac{\partial c}{\partial x} \frac{\partial c}{\partial x} - \frac{1}{2} (\text{grad } c)^2, \quad ckX_y = \frac{\partial c}{\partial x} \frac{\partial c}{\partial y}, \quad ckX_z = \frac{\partial c}{\partial x} \frac{\partial c}{\partial z} \quad \text{etc.} \end{array} \right.$$

Thus, the reaction principle is indeed satisfied. The term added to equation (3b) in order to satisfy the reaction principle wins our confidence thanks to the following argument.

If each and every energy density (σc) produces a (negative) divergence of the gravitational lines of force, then this must also hold for the energy density of gravitation itself. If one writes (3b) in the form

$$\Delta c = k \left\{ c\sigma + \frac{1}{2k} \frac{\text{grad}^2 c}{c} \right\},$$

one immediately sees that the second term within the bracket is to be viewed as the energy density of the gravitational field.⁴ It remains now to show that only this term denotes the energy density of the gravitational field according to the energy principle as well.

To that end, we imagine a spatial arrangement of ponderable masses (density σ) situated in a finite region and enclosed by an infinitely distant surface; let c tend to a constant value at infinity, insofar as this is permitted by equations (3b) and (3b'). We then have to prove that the work δA to be supplied to the system for an arbitrary infinitesimal displacement of the masses (δx , δy , δz) is equal to the increase δE of the integral (which is extended over the entire space) of the total energy density, which is given within the brackets in the above equation.

One first obtains by virtue of (4)

[29] ⁴It has to be pointed out that it attains a positive value, as with Abraham.