# Does the region inside the event horizon of a black hole consist of a single spatial point ?

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**Abstract**

It will be proposed here that the region inside the event horizon of a black hole should be considered as consisting of a single spatial point.

## 1 Introduction

In Schwarzschild coordinates, the metric outside the surface of a static and spherically symmetric local distribution of matter with total mass-energy $M$ is given by[1]:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{1}$$

Using the Schwarzschild radius $R_S$ defined by :

$$R_S = 2M \tag{2a}$$

we can write (1) as :

$$ds^2 = -\left(1 - \frac{R_S}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{R_S}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{2b}$$

The question arises here : what is the appropriate range of values for $r$ when the total mass-energy $M$ is entirely contained in the region defined by $r \le R_S$ ?

From a mathematical point of view, we can assign to $r$ the same range of values than the one used for a total mass-energy $M$ not entirely located within the region defined by $r \le R_S$, that is : $0 \le r \le \infty$. But from a physical point of view, in virtue of General Relativity the only values of coordinates that have a physical meaning are those corresponding to observable physical events, or that can be made so. For a total mass-energy $M$ entirely located whithin the region defined by $r \leq R_S$ , it is well known that events located within the region defined by $r < R_S$ are not observable for an observer located outside that region. The surface of this region, known as the "event horizon", has a not null area $A_S$ given by :

$$A_S = 4\pi R_S{}^2 \tag{3}$$

Does the not null value for $A_S$ legitimate one to conclude that the spatial region inside the surface $r = R_S$ consists of more than one single spatial point ? It will be argued here that this is not the case, that is to say, all the spatial points within the region defined by $r < R_S$ can be considered as corresponding to a single spatial point and therefore the appropriate range of values for $r$ would be $R_S \leq r \leq \infty$ .

## 2 A simple case : a curved two-dimensional space

Let us consider a two-dimensional space $S$ . In the case of a flat geometry, the distance $ds$ between two neighbour points $P_1$ and $P_2$ is given in polar coordinates by :

$$ds^2 = dr^2 + r^2 d\theta^2 \tag{4}$$

It is important to notice here that an infinite number of coordinates pairs $(0,\theta)$, with $0 \leq \theta \leq 2\pi$ , correspond to a single point, that is, the origin of the coordinates.

Now let us consider a curved space $S$ with a metric that is rotationnaly symmetric about some point $P_0$, that is, the metric in polar coordinates can be written as :

$$ds^2 = F(r)^2 dr^2 + G(r)^2 r^2 d\theta^2 \tag{5}$$

where $F(r)$ and $G(r)$ are arbitrary continuous functions.

Let us take :

$$F(r) = \left(1 + \frac{R_S}{r}\right)^{1/2} \tag{6a}$$

$$G(r) = \left(1 + \frac{R_S}{r}\right) \tag{6b}$$

where $R_S$ is an arbitrary not null positive number.

The metric (5) is therefore given by :

$$ds^2 = \left(1 + \frac{R_S}{r}\right) dr^2 + \left(1 + \frac{R_S}{r}\right)^2 r^2 d\theta^2 \qquad (6c)$$

The circumference $L$ of a circle $C$ centered on $P_0$ and passing through some point $P_1$ with coordinates ( $r_1$ , $\theta$) is :

$$L = \int_0^{2\pi} \left(1 + \frac{R_S}{r_1}\right) r_1 d\theta = 2\pi r_1 + 2\pi R_S \qquad (7)$$

If we take the limit $P_1 \to P_0$, that is, $r_1 \to 0$ , we get :

$$L_0 \equiv \lim_{r_1 \to 0} L = \lim_{r_1 \to 0} (2\pi r_1 + 2\pi R_S) = 2\pi R_S \qquad (8)$$

That is to say, the circle $C_0$ defined symbolically as :

$$C_0 \equiv \lim_{r_1 \to 0} C \qquad (9)$$

has a circumference different of zero and contains the only point $P_0$.

The same kind of reasoning in a three-dimensional space leads to the result that the region inside a surface having an area different from zero can consist of a single point.

Therefore, the answer to the question aroused in the Introduction :
" Does the not null value for $A_S$ legitimate one to conclude that the spatial region inside the surface $r = R_S$ consists of more than one single spatial point ? "
is clearly negative.

## 3 The event horizon of a black hole

Let us now turn back to the event horizon of a black hole. We know from General Relativity that all coordinates systems are on an equal footing to describe the physical world. We could proceed with Schwarzschild coordinates but the point will possibly appear more clear with a new set of coordinates. Let us then perform the following continuous transformation of coordinates on Schwarzschild coordinates :

$$r' = r - 2M \qquad (10a)$$
$$\theta' = \theta \qquad (10b)$$
$$\varphi' = \varphi \qquad (10c)$$
$$t' = t \qquad (10d)$$

With these new coordinates, the metric specified by (1) is given by :

$$ds^2 = -\frac{1}{\left(1+\frac{R_S}{r'}\right)} dt^2 + \left(1+\frac{R_S}{r'}\right) dr'^2 + \left(1+\frac{R_S}{r'}\right)^2 r'^2 \left(d\theta'^2 + \sin^2\theta' \, d\varphi'^2\right) \qquad (11)$$

This metric is well-behaved everywhere in the region defined by $r' > 0$ .

The spatial part of *ds* in (11) is the three-dimensional analog of (6c). Considering the discussion following (6c), the answer to the following question : "Does the region inside the surface defined by $r = R_S$ reduce to a single spatial point ?" can clearly be positive. Moreover, if we take into account the argument presented in the Introduction concerning the fact that any event interior to that region is unobservable for an observer located outside that region (that is, for us), it clearly appears more appropriate to consider that the region inside the event horizon of a black hole contains a single spatial point, with the metric being singular at that point. The range of values for *r'* is thus $0 \le r' \le \infty$ and the mass-energy distribution must then be expressed as a volume mass-energy density given in rectangular coordinates (*x',y',z'*) by:

$$\rho(x', y', z') = M \; \delta(x')\delta(y')\delta(z') \qquad (12a)$$

For all $r' > 0$, the metric given by (11) is the solution of Eintein's equation for the spherically symmetric mass-energy distribution given by (12a) with the usual boundary conditions and as required we have $m(r')=M$ for all $r'>0$, where $m(r')$ is the mass-energy inside radius *r'*.

Turning back to the the Schwarzschild coordinates with the *r* coordinate now restricted to $r \ge R_S$ , the mass-energy density in (12a) becomes a surface mass-energy density which is not null only on the surface defined by $r = R_S$ and is given by:

$$\sigma(\theta,\varphi) = \frac{M}{4\pi R_S^2} \qquad (12b)$$

Taking this surface mass-energy density and restricting the *r* coordinate to $r \ge R_S$ , we can perform the same calculations as the ones performed in MTW[1] (Chapter 23) : this statement should however be confirmed by other authors before being considered as a valid one. All the known observable phenomena associated with black holes remain applicable given that any such observable phenomenon is initiated on or outside the event horizon.

## 4 Conclusion

The analysis presented in the above shows that it appears more appropriate to consider that the region inside the event horizon of a black hole consists of a single spatial point (more precisely a world-line specified by $(x_0, y_0, z_0, t)$ with $x_0, y_0, z_0$ kept fixed) with the singularity located at that point. The singularity at the center of a black hole is then much like the usual $r = 0$ singularity with the difference that the surface just outside $r = 0$ is not null and is given by $16\pi M^2$.

## References

[1] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation*, W.H. Freeman and company, 1973