A static black hole model with no unobservable region surrounding the central singularity

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Abstract

A static black hole model with no unobservable region surrounding the central singularity is proposed here. In this model, the singularity of the black hole is thus much like the usual singularity at $r = 0$ for point-like objects, the main difference being the minimum value of $16\pi M^2$ for a surface surrounding the singularity.

1 Introduction

In Schwarzschild coordinates, the metric outside the surface of a static and spherically symmetric local distribution of matter with total mass-energy M is given by[1]:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{1}$$

Using the Schwarzschild radius R_S defined by :

$$R_S = 2M \tag{2a}$$

equation (1) can be written as :

$$ds^2 = -\left(1 - \frac{R_S}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{R_S}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{2b}$$

The question arises here : what is the appropriate range of values for r when the total mass-energy M is entirely contained within the region defined by $r \le R_S$?

From a mathematical point of view, we can assign to r the same range of values than the one used for a total mass-energy M not entirely located within the region defined by $r \le R_S$, that is : $0 \le r \le \infty$. But from a physical point of view, according to General

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Relativity the only values of coordinates that have a physical meaning are those corresponding to observable physical events, or that can be made so. For a total mass-energy M entirely located whithin the region defined by $r \leq R_S$, it is well known that events located within this region are not observable for an observer located outside that region. The surface of this region, known as the "event horizon", has a not null area A_S given by :

$$A_S = 4\pi R_S{}^2 \qquad (3)$$

The usual interpretation is that the r coordinate becomes a timelike coordinate in the range $0 \leq r < R_S$, with any event in that region being unobservable for an observer located outside that region. It will be shown here that one can construct a black hole model by choosing a radial r' coordinate different from the Schwarzschild's r coordinate. This new radial coordinate is a spacelike coordinate in the whole range $0 \leq r' \leq \infty$, with all the events for any value $r' > 0$ being observable. The black hole's central singularity is located at $r' = 0$ as usual.

2 A property of curved three-dimensional space

Let us consider a three-dimensional space S . In the case of a flat geometry, the distance ds between two neighbour points P_1 and P_2 is given in spherical coordinates by :

$$ds^2 = dr^2 + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right) \qquad (4)$$

Now let us consider a curved space S with a metric that is spherically symmetric about some point P_0, that is, the metric in spherical coordinates can be written as :

$$ds^2 = F(r)^2 dr^2 + G(r)^2 r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right) \qquad (5)$$

where $F(r)$ and $G(r)$ are arbitrary continuous functions.

Let us take :

$$F(r) = \left(1 + \frac{R_S}{r}\right)^{1/2} \qquad (6a)$$

$$G(r) = \left(1 + \frac{R_S}{r}\right) \qquad (6b)$$

where R_S is an arbitrary not null positive number.

The metric (5) is therefore given by :

$$ds^2 = \left(1+\frac{R_S}{r}\right)dr^2 + \left(1+\frac{R_S}{r}\right)^2 r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (6c)$$

The area A of a spherical surface S centered on P_0 and containing some point P_1 with coordinates (r_1, θ, φ) is :

$$A = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(1+\frac{R_S}{r_1}\right)^2 r_1^{\,2}\sin\theta\, d\varphi\, d\theta = 4\pi(r_1 + R_S)^2 \qquad (7)$$

If we take the limit $P_1 \to P_0$, that is, $r_1 \to 0$, we get :

$$A_0 \equiv \lim_{r_1\to 0} A = \lim_{r_1\to 0}\left[4\pi\left(r_1 + R_S\right)^2\right] = 4\pi R_S^{\,2} \qquad (8)$$

That is to say, the area A of a spherical surface S centered on P_0 has a minimum value A_{min} given :

$$A_{min} = A_0 = 4\pi R_S^{\,2} \qquad (9)$$

We thus see that in a curved spatial geometry, the minimum area of a spherical surface centered on a given point can be a not null value.

3 A static black hole model

Let us now turn back to black holes. We know from General Relativity that all coordinates systems are on an equal footing to describe the physical world. Let us then perform the following continuous transformation of coordinates on Schwarzschild coordinates :

$$r' = r - R_S \qquad (10a)$$

$$\theta' = \theta \qquad (10b)$$

$$\varphi' = \varphi \qquad (10c)$$

$$t' = t \qquad (10d)$$

The range for the Schwarzschild r-coordinate is $0 \le r \le \infty$. Therefore the range for r' is $-R_S \le r' \le \infty$.

With these new coordinates, the metric specified by (1) is given by :

$$ds^2 = -\frac{1}{\left(1+\frac{R_S}{r'}\right)}dt'^2 + \left(1+\frac{R_S}{r'}\right)dr'^2 + \left(1+\frac{R_S}{r'}\right)^2 r'^2\left(d\theta'^2 + \sin^2\theta'\, d\varphi'^2\right) \qquad (11)$$

It is to be noticed here that the spatial part of ds in (11) is identical to the three-dimensional spatial metric (6c) and thus the minimum area of any spherical surface surrounding $r'=0$ has a minimum value equal to $4\pi R_S{}^2$, or equivalently $16\pi M^2$.

As mentioned in the Introduction, only the events with $r > R_S$ are observable for an observer located in that region. Let us then choose $0 \le r' \le \infty$ as the new range of values for r'. With this new range of values for r', we must prove that we can find a new mass-energy distribution, with total mass-energy *M*, for which the metric (11) is the unique solution of field's equations in the region $0 \le r' \le \infty$.

Let us take the following spherically symmetric mass-energy density with total mass-energy equal to *M* :

$$\rho(r',\theta',\varphi') = \frac{M r'^{1/2}}{4\pi R_S{}^{5/2}}\,\delta(r') \qquad (12)$$

In order to prove that the mass-energy density in (12) leads to the metric (11), we turn back to the Schwarzschild coordinates but now with $2M \le r \le \infty$ as the new range of values for r. The mass-energy density (12) expressed in Schwarzschild coordinates is given by:

$$\rho(r,\theta,\varphi) = \frac{M\left(r-R_S\right)^{1/2}}{4\pi R_S{}^{5/2}}\,\delta(r-R_S) \qquad (13)$$

The way in which field's equations are solved in Schwarzschild coordinates for a spherically symmetric mass-energy distribution is given in MTW[1] (Chapter 23). We have the only difference here that the range of values for r is $r \ge R_S$ instead of $r \ge 0$. We must therefore integrate from $r = R_S$ instead of $r = 0$ in equations (23.19)[1] and (23.21)[1]. In addition, we must replace the condition $m(0) = 0$ by $m(R_S) = 0$, where $m(r)$ is "the mass-energy inside radius r". Given that the region $r > R_S$ is outside the "surface of the star", the metric given by (1) with $r > R_S$ is therefore the unique solution of the field's equations for the mass-energy density given in (13). Therefore (11) is the unique solution in coordinates (r',θ',φ',t') of the field's equations for the mass-energy density given in (12), with $0 \leq r' \leq \infty$.

4 Which is the appropriate range of values for the radial coordinate ?

For a static black hole, Schwarzschild metric (1) is the unique solution to field's equations for the range $0 \leq r \leq \infty$, with the mass-energy density equal to zero everywhere except at $r = 0$. In the same way, the metric (11) is the unique solution to field's equations for the range $0 \leq r' \leq \infty$, with the mass-energy density equal to zero everywhere except at $r' = 0$. Which of r or r' is the appropriate radial coordinate ? The answer to this question will be known only when we will be able to provide a detailed description of the transition from an "ordinary" star to a black hole.

5 Conclusion

The analysis presented in the above shows that it is possible to have a black hole's model without an unobservable region surrounding the central singularity. With such a model, the singularity at the center of a black hole is then much like the usual $r = 0$ singularity with the difference that any spherical surface centered on $r = 0$ has a minimum value given by $16\pi M^2$. All the known results concerning observable phenomena associated with black holes remain applicable given that any such observable phenomenon is initiated on or outside the event horizon.

The proof that such a model holds true for more general black holes is postponed to later.

References

[1] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation,* W.H. Freeman and company, 1973