

Phénomènes de Transferts

Examen Final – 11/06/2015

Exercice 1

On s'intéresse à une colonne en béton à l'extérieur d'un bâtiment. Cette colonne, de diamètre $D = 30$ cm et de hauteur $H = 3.5$ m se trouve au petit matin à une température uniforme de $T_i = 16^\circ\text{C}$. Elle est alors chauffée par l'air ambiant, dont la vitesse est d'environ $V_{air} = 3$ m/s et la température moyenne $T_{air} = 28^\circ\text{C}$.

Déterminer le temps qu'il faudra attendre pour que la surface de la colonne atteigne une température de $T_S = 27^\circ\text{C}$.

Indication : Avant de se jeter sur une formule inappropriée, on veillera à se calmer, à prendre une grande respiration et à bien réfléchir aux différentes étapes de la méthode qui vous amènera à une réponse réaliste.

Exercice 2

Une personne en tenue légère se sent à l'aise dans une pièce thermalisée à $T_a = 22^\circ\text{C}$, et dont les parois sont également à température $T_p = 22^\circ\text{C}$.

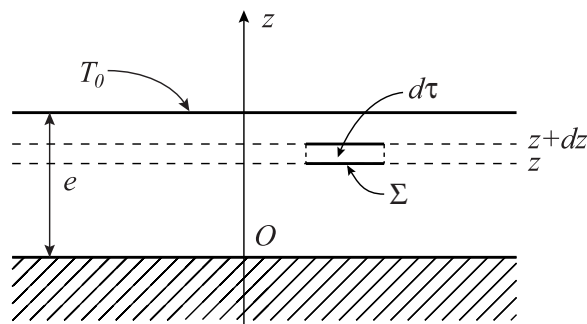
Par un jour de grand froid, la température des parois de la pièce tombe à $T'_p = 18^\circ\text{C}$.

À quelle température T'_a faut-il monter l'air de la pièce pour que la personne se sente aussi bien dans la même tenue ?

On considèrera que l'émissivité de la personne est $\varepsilon = 0.95$, et que les mouvements d'air dans la pièce se font à une vitesse de l'ordre de 0.2 m/s.

Exercice 3

Un revêtement métallique d'épaisseur e est parcouru par un courant électrique d'intensité constante et uniforme. Ce courant produit, par effet Joule, une densité volumique q d'énergie thermique supposée uniforme dans tout le revêtement. La face supérieure du revêtement est maintenue à la température T_0 , tandis que la face inférieure est isolée thermiquement.



La couche est supposée assez large dans les directions transversales pour que la température ne dépende que de la variable z .

1. Exprimer le flux de chaleur Φ_Σ à travers une surface Σ du plan horizontal d'ordonnée z , dans la direction des z positifs, en fonction de k , Σ et du profil de température $T(z)$.
2. Effectuer un bilan thermique pour le volume élémentaire dV , compris entre les éléments de surface Σ_z et Σ_{z+dz} d'ordonnées z et $z + dz$ (cf schéma ci-dessus).

3. En déduire l'équation vérifiée par la distribution de température $T(z)$.
4. Écrire les conditions aux limites en $z = 0$ et $z = e$.
5. Donner alors la distribution de température $T(z)$, et calculer le flux surfacique sortant à travers la face supérieure du revêtement.

Données :

- Constante de Stéphan-Boltzmann : $\sigma = 5,67 \cdot 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$
- Masse volumique du béton : $\rho_b = 1600 \text{ kg/m}^3$
- Capacité calorifique massique du béton : $c_b = 840 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
- Diffusivité thermique du béton : $\alpha_b = 5.9 \cdot 10^{-7} \text{ m}^2/\text{s}$

Pour l'air, on prendra (à $\simeq 25^\circ\text{C}$) :

- Diffusivité thermique : $\alpha_{air} = 22 \cdot 10^{-6} \text{ m}^2/\text{s}$
- Conductivité thermique : $k_{air} = 0.026 \text{ W} / \cdot \text{m}^{-1} \cdot \text{K}^{-1}$
- Viscosité cinématique : $\nu_{air} = 16 \cdot 10^{-6} \text{ m}^2/\text{s}$
- Des éléments de référence concernant les diagrammes de Heissler et les corrélations pour la convection forcée sont donnée ci-après. (d'après Y. A. Cengel, "Heat and Mass Transfer : Fundamentals and Applications", ISBN : 0073398128.)

SUMMARY

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\text{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\text{Re}_{x,cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

$$\text{Laminar: } C_{f,x} = \frac{0.664}{\text{Re}_x^{1/2}} \quad \text{Re}_x < 5 \times 10^5$$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.332 \text{Re}_x^{0.5} \text{Pr}^{1/3} \quad \text{Pr} > 0.6$$

$$\text{Turbulent: } C_{f,x} = \frac{0.0592}{\text{Re}_x^{1/5}}, \quad 5 \times 10^5 \leq \text{Re}_x \leq 10^7$$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296 \text{Re}_x^{0.8} \text{Pr}^{1/3} \quad \begin{matrix} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_x \leq 10^7 \end{matrix}$$

The *average friction coefficient* relations for flow over a flat plate are:

$$\text{Laminar: } C_f = \frac{1.328}{\text{Re}_L^{1/2}} \quad \text{Re}_L < 5 \times 10^5$$

$$\text{Turbulent: } C_f = \frac{0.074}{\text{Re}_L^{1/5}} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7$$

$$\text{Combined: } C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7$$

$$\text{Rough surface, turbulent: } C_f = \left(1.89 - 1.62 \log \frac{x}{L}\right)^{-2.5}$$

The average Nusselt number relations for flow over a flat plate are:

$$\text{Laminar: } \text{Nu} = \frac{hL}{k} = 0.664 \text{Re}_L^{0.5} \text{Pr}^{1/3} \quad \text{Re}_L < 5 \times 10^5$$

Turbulent:

$$\text{Nu} = \frac{hL}{k} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} \quad \begin{matrix} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{matrix}$$

Combined:

$$\text{Nu} = \frac{hL}{k} = (0.037 \text{Re}_L^{0.8} - 871) \text{Pr}^{1/3}, \quad \begin{matrix} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{matrix}$$

For isothermal surfaces with an unheated starting section of length ξ , the local Nusselt number and the average convection coefficient relations are

$$\text{Laminar: } \text{Nu}_x = \frac{\text{Nu}_{x(\text{for } \xi=0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \frac{0.332 \text{Re}_x^{0.5} \text{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$$

$$\text{Turbulent: } \text{Nu}_x = \frac{\text{Nu}_{x(\text{for } \xi=0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \frac{0.0296 \text{Re}_x^{0.8} \text{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$$

$$\text{Laminar: } h = \frac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L} h_{x=L}$$

$$\text{Turbulent: } h = \frac{5[1 - (\xi/x)^{9/10}]}{(1 - \xi/L)} h_{x=L}$$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

$$\begin{aligned} \text{Laminar:} \quad \text{Nu}_x &= 0.453 \text{Re}_x^{0.5} \text{Pr}^{1/3} \\ \text{Turbulent:} \quad \text{Nu}_x &= 0.0308 \text{Re}_x^{0.8} \text{Pr}^{1/3} \end{aligned}$$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}} \left[1 + \left(\frac{\text{Re}}{282,000} \right)^{5/8} \right]^{4/5}$$

which is valid for $\text{Re Pr} > 0.2$, and

$$\text{Nu}_{\text{sph}} = \frac{hD}{k} = 2 + [0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3}] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s} \right)^{1/4}$$

which is valid for $3.5 \leq \text{Re} \leq 80,000$ and $0.7 \leq \text{Pr} \leq 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s , which is evaluated at the surface temperature T_s) in the case of a sphere.

In tube banks, the Reynolds number is based on the maximum velocity \mathcal{V}_{max} that is related to the approach velocity \mathcal{V} as

In-line and *Staggered* with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\text{max}} = \frac{S_T}{S_T - D} \mathcal{V}$$

Staggered with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\text{max}} = \frac{S_T}{2(S_D - D)} \mathcal{V}$$

where S_T the transverse pitch and S_D is the diagonal pitch. The average Nusselt number for cross flow over tube banks is expressed as

$$\text{Nu}_D = \frac{hD}{k} = C \text{Re}_D^m \text{Pr}^n (\text{Pr}/\text{Pr}_s)^{0.25}$$

where the values of the constants C , m , and n depend on value Reynolds number. Such correlations are given in Table 7-2. All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid defined as $T_m = (T_i + T_o)/2$.

The average Nusselt number for tube banks with less than 16 rows is expressed as

$$\text{Nu}_{D,N_L} = F \text{Nu}_D$$

where F is the *correction factor* whose values are given in Table 7-3. The heat transfer rate to or from a tube bank is determined from

$$\dot{Q} = hA_s \Delta T_{\text{ln}} = \dot{m} C_p (T_e - T_i)$$

where ΔT_{ln} is the logarithmic mean temperature difference defined as

$$\Delta T_{\text{ln}} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

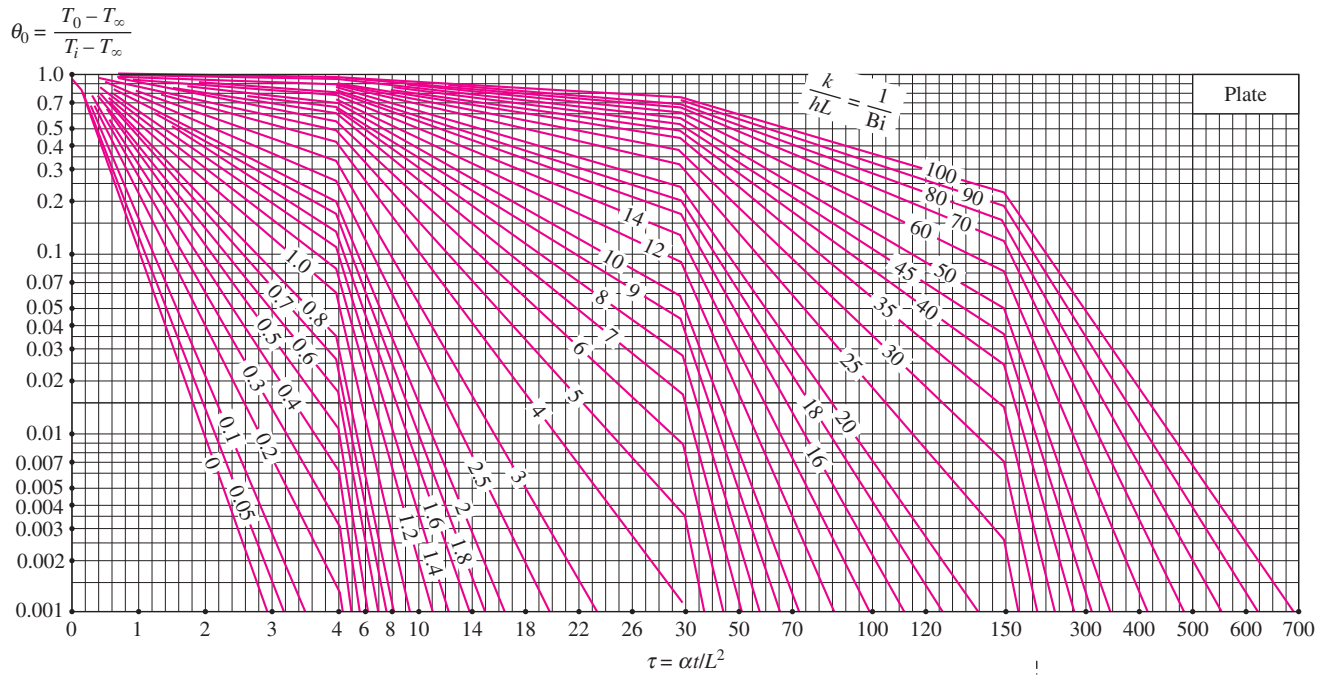
and the exit temperature of the fluid T_e is

$$T_e = T_s - (T_s - T_i) \exp\left(\frac{A_s h}{\dot{m} C_p}\right)$$

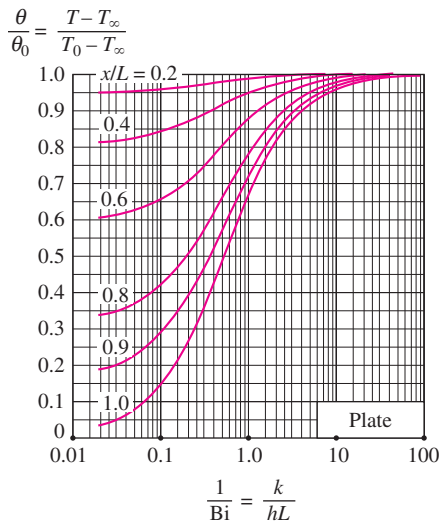
where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho \mathcal{V} (N_T S_T L)$ is the mass flow rate of the fluid. The pressure drop ΔP for a tube bank is expressed as

$$\Delta P = N_L f \chi \frac{\rho \mathcal{V}_{\text{max}}^2}{2}$$

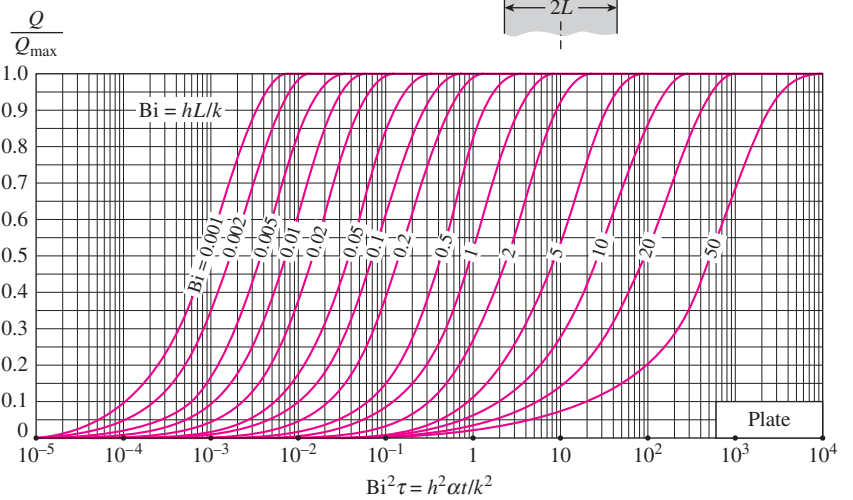
where f is the friction factor and χ is the correction factor, both given in Figs. 7-27.



(a) Midplane temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227–36. Reprinted by permission of ASME International.)

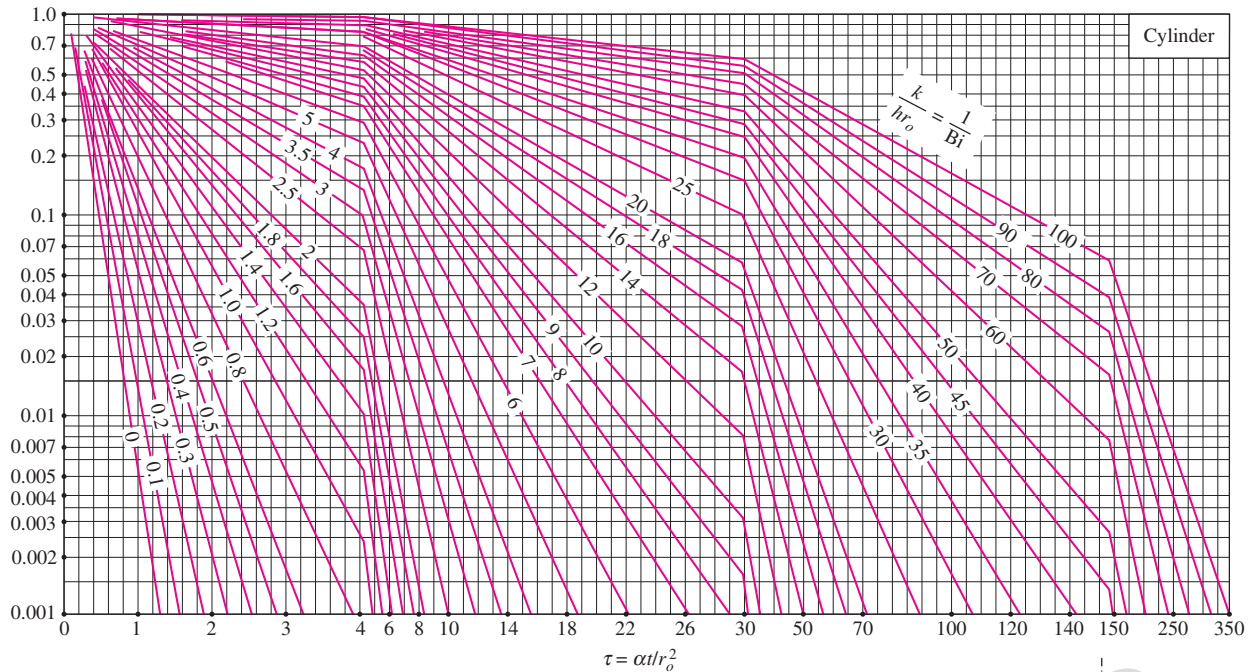


(b) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227–36. Reprinted by permission of ASME International.)



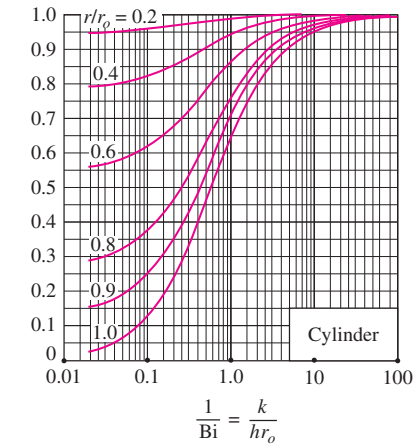
(c) Heat transfer (from H. Gröber et al.)

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty}$$



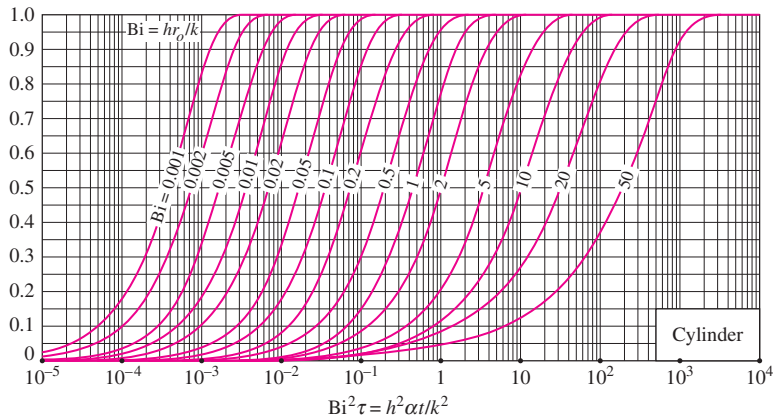
(a) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227-36. Reprinted by permission of ASME International.)

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty}$$



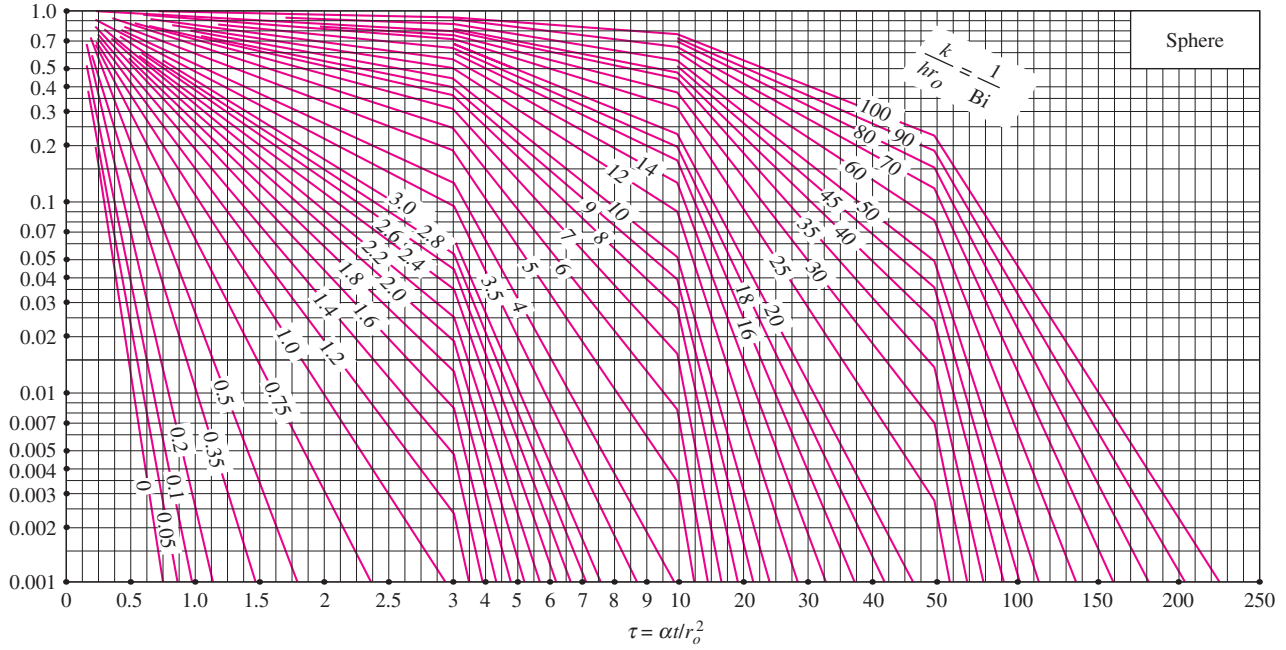
(b) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227-36. Reprinted by permission of ASME International.)

$$\frac{Q}{Q_{\max}}$$

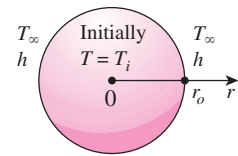


(c) Heat transfer (from H. Gröber et al.)

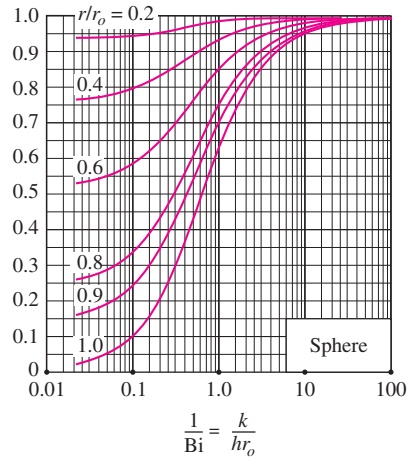
$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty}$$



(a) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227–36. Reprinted by permission of ASME International.)

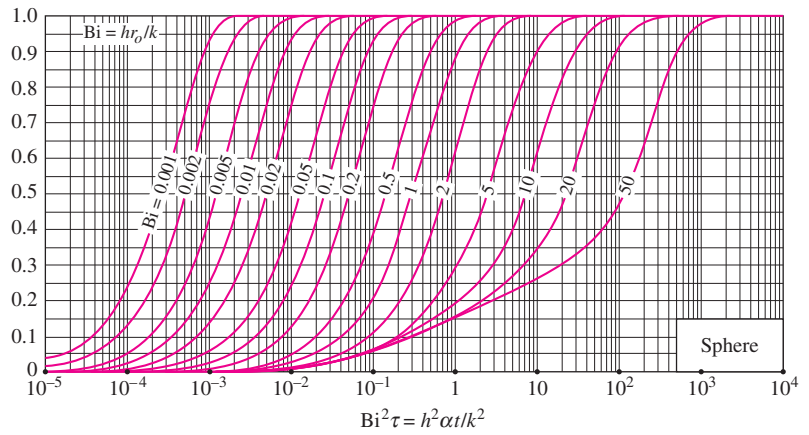


$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty}$$



(b) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME* 69, 1947, pp. 227–36. Reprinted by permission of ASME International.)

$$\frac{Q}{Q_{\max}}$$



(c) Heat transfer (from H. Gröber et al.)