On veut prouver que :

$$\left\{\begin{array}{l} (\vec{v}\cdot\overrightarrow{\text{grad}})\cdot\vec{v} = \overrightarrow{\text{grad}}\left(\frac{v^2}{2}\right) + (\overrightarrow{\text{rot}}\,\vec{v})\wedge\vec{v} \\ (\vec{v}\cdot\vec{\nabla})\cdot\vec{v} = \vec{\nabla}\left(\frac{v^2}{2}\right) + (\vec{\nabla}\wedge\vec{u})\wedge\vec{u} \end{array}\right.$$

Nabla

$$\left|\begin{array}{l} \vec{v} = v_x\vec{e_x} + v_y\vec{e_y} + v_z\vec{e_z} \\ \vec{\nabla} = \frac{\partial}{\partial x}\vec{e_x} + \frac{\partial}{\partial y}\vec{e_y} + \frac{\partial}{\partial z}\vec{e_z} \end{array}\right.$$

- $$\vec{v}\cdot\vec{\nabla} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}\cdot\begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} = v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}$$

$$\hookrightarrow (\vec{v}\cdot\overrightarrow{\text{grad}})\vec{v} = \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$(\vec{v}\cdot\overrightarrow{\text{grad}})\cdot\vec{v} = \begin{pmatrix} v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} \\ v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z} \\ v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z} \end{pmatrix}$$

- 2nd membre de l'equation :

On a : $\frac{v^2}{2} = \frac{v_x^2}{2} + \frac{v_y^2}{2} + \frac{v_z^2}{2}$ (scalaire)

Donc $$\vec{\nabla}\left(\frac{v^2}{2}\right) = \begin{pmatrix} \frac{\partial(v^2/2)}{\partial x} \\ \frac{\partial(v^2/2)}{\partial y} \\ \frac{\partial(v^2/2)}{\partial z} \end{pmatrix} = \begin{pmatrix} v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_y}{\partial x} + v_z\frac{\partial v_z}{\partial x} \\ v_x\frac{\partial v_x}{\partial y} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_z}{\partial y} \\ v_x\frac{\partial v_x}{\partial z} + v_y\frac{\partial v_y}{\partial z} + v_z\frac{\partial v_z}{\partial z} \end{pmatrix}$$

Et $$\vec{\nabla}\wedge\vec{u} = \begin{pmatrix} \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \\ \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \\ \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \end{pmatrix} \Rightarrow \sout{(\vec{\nabla}\wedge\vec{u})\wedge\vec{u} =}$$

~~$u_y\frac{\partial u_x}{\partial z} - u_y\frac{\partial u_z}{\partial x} - u_y\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_x}{\partial y}$~~