

$$(\vec{\nabla} \wedge \vec{v}) \wedge \vec{v} = \begin{pmatrix} \frac{\partial}{\partial z} v_x v_z - \frac{\partial}{\partial x} v_z v_x - \frac{\partial}{\partial x} v_y v_y + \frac{\partial}{\partial y} v_x v_y \\ \frac{\partial}{\partial x} v_y v_x - \frac{\partial}{\partial y} v_x v_x - \frac{\partial}{\partial y} v_z v_z + \frac{\partial}{\partial z} v_y v_z \\ \frac{\partial}{\partial y} v_z v_y - \frac{\partial}{\partial z} v_y v_y - \frac{\partial}{\partial z} v_x v_x + \frac{\partial}{\partial x} v_z v_x \end{pmatrix}$$

$$\text{Donc } (\vec{\nabla} \wedge \vec{v}) \wedge \vec{v} + \vec{\nabla} \left( \frac{v^2}{2} \right) = \begin{pmatrix} v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_z}{\partial z} \\ v_y \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_y \frac{\partial v_z}{\partial z} \\ v_z \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{pmatrix}$$

On a donc 
$$\boxed{(\vec{v} \cdot \vec{\nabla}) \cdot \vec{v} = (\vec{\nabla} \wedge \vec{v}) \wedge \vec{v} + \vec{\nabla} \left( \frac{v^2}{2} \right)}$$

$$(\vec{v} \cdot \vec{\nabla}) \cdot \vec{v} \stackrel{?}{=} \vec{\nabla}(\vec{u}) \cdot \vec{u}$$

gradient d'un vecteur  $\Rightarrow$  Matrice  $3 \times 3$

$$\vec{\nabla}(\vec{u}) = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$$\vec{\nabla}(\vec{u}) \cdot \vec{u} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$= \begin{bmatrix} v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_y}{\partial y} + v_x \frac{\partial v_z}{\partial z} \\ v_y \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_y \frac{\partial v_z}{\partial z} \\ v_z \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{bmatrix} \neq (\vec{v} \cdot \vec{\nabla}) \cdot \vec{v}$$