Hence

$$D_x = D_y = \frac{Eh^3}{12(1-\nu^2)}$$

$$H = D_1 + 2D_{xy} = \frac{h^3}{12}\left(\frac{\nu E}{1-\nu^2} + \frac{E}{1+\nu}\right) = \frac{Eh^3}{12(1-\nu^2)} \qquad (f)$$

and Eq. (213) reduces to our previous Eq. (103).

Equation (213) can be used in the investigation of the bending of plates of nonisotropic and even nonhomogeneous material, such as reinforced concrete slabs,[1] which has different flexural rigidities in two mutually perpendicular directions.

86. Determination of Rigidities in Various Specific Cases. The expressions (d) given for the rigidities in the preceding article are subject to slight modifications according to the nature of the material employed. In particular, all values of torsional rigidity D_{xy} based on purely theoretical considerations should be regarded as a first approximation, and a direct test as shown in Fig. 25c must be recommended in order to obtain more reliable values of the modulus G. Usual values of the rigidities in some cases of practical interest are given below.

Reinforced Concrete Slabs. Let E_s be Young's modulus of steel, E_c that of the concrete, ν_c Poisson's ratio for concrete, and $n = E_s/E_c$. In terms of the elastic constants introduced in Art. 85 we have approximately $\nu_c = E''/\sqrt{E'_x E'_y}$. For a slab with two-way reinforcement in the directions x and y we can assume

$$D_x = \frac{E_c}{1-\nu_c^2}[I_{cx} + (n-1)I_{sx}]$$

$$D_y = \frac{E_c}{1-\nu_c^2}[I_{cy} + (n-1)I_{sy}] \qquad (a)$$

$$D_1 = \nu_c\sqrt{D_x D_y}$$

$$D_{xy} = \frac{1-\nu_c}{2}\sqrt{D_x D_y}$$

In these equations, I_{cx} is the moment of inertia of the slab material, I_{sx} that of the reinforcement taken about the neutral axis in the section x = constant, and I_{cy} and I_{sy} are the respective values for the section y = constant.

With the expression given for D_{xy} (also recommended by Huber) we obtain

$$H = \sqrt{D_x D_y} \qquad (b)$$

and the differential equation

$$D_x\frac{\partial^4 w}{\partial x^4} + 2\sqrt{D_x D_y}\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} = q \qquad (c)$$

[1] The application of the theory of anisotropic plates to reinforced concrete slabs is due to M. T. Huber, who published a series of papers on this subject; see *Z. Österr. Ing. u. Architektur Ver.*, 1914, p. 557. The principal results are collected in his books: "Teorya Plyt," Lvov, 1922, and "Probleme der Statik technisch wichtiger orthotroper Platten," Warsaw, 1929. Abstracts of his papers are given in *Compt. rend.*, vol. 170, pp. 511 and 1305, 1920; and vol. 180, p. 1243, 1925.

which can readily be reduced to the form (103) by introducing $y_1 = y\sqrt[4]{D_x/D_y}$ as a new variable.

It is obvious that the values (a) are not independent of the state of the concrete. For instance, any difference of the reinforcement in the directions x and y will affect the ratio D_x/D_y much more after cracking of the concrete than before.

Plywood. For a plate glued together of three or five plies, the x axis supposed to be parallel to the face grain, we may use the constants given in Table 79.

TABLE 79. ELASTIC CONSTANTS FOR PLYWOOD
Unit = 10^6 psi

Material	E'_x	E'_y	E''	G
Maple,* 5-ply	1.87	0.60	0.073	0.159
Afara,* 3-ply	1.96	0.165	0.043	0.110
Gaboon* (Okoumé), 3-ply	1.28	0.11	0.014	0.085
Birch,† 3- and 5-ply	2.00	0.167	0.077	0.17
Birch† with bakelite membranes	1.70	0.85	0.061	0.10

* By R. F. S. Hearmon and E. H. Adams, *Brit. J. Appl. Phys.*, vol. 3, p. 155, 1952.
† By S. G. Lechnitzky, "Anisotropic Plates," p. 40, Moscow, 1947.

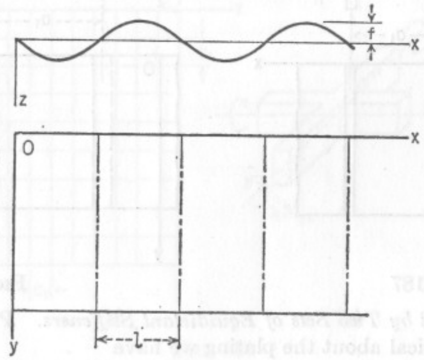

FIG. 186

Corrugated Sheet. Let E and ν be the elastic constants of the material of the sheet, h its thickness,

$$z = f\sin\frac{\pi x}{l}$$

the form of the corrugation, and s the length of the arc of one-half a wave (Fig. 186). Then we have[1]

$$D_x = \frac{l}{s}\frac{Eh^3}{12(1-\nu^2)}$$

$$D_y = EI$$

$$D_1 \sim 0$$

$$H = 2D_{xy} = \frac{s}{l}\frac{Eh^3}{12(1+\nu)}$$

[1] See E. Seydel, *Ber. deut. Versuchsanstalt Luftfahrt*, 1931.

in which, approximately,

$$s = l\left(1 + \frac{\pi^2 f^2}{4l^2}\right)$$

$$I = \frac{f^2 h}{2}\left[1 - \frac{0.81}{1 + 2.5\left(\frac{f}{2l}\right)^2}\right]$$

Plate Reinforced by Equidistant Stiffeners in One Direction. For a plate reinforced symmetrically with respect to its middle plane, as shown in Fig. 187, we may take[1]

$$D_x = H = \frac{Eh^3}{12(1-\nu^2)}$$

$$D_y = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I}{a_1}$$

in which E and ν are the elastic constants of the material of the plating, E' the Young modulus, and I the moment of inertia of a stiffener, taken with respect to the middle axis of the cross section of the plate.

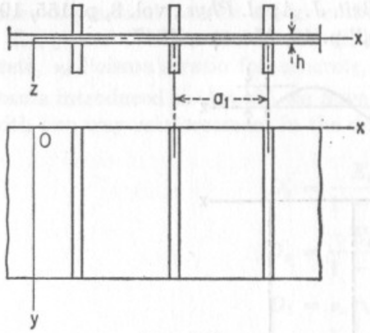

FIG. 187

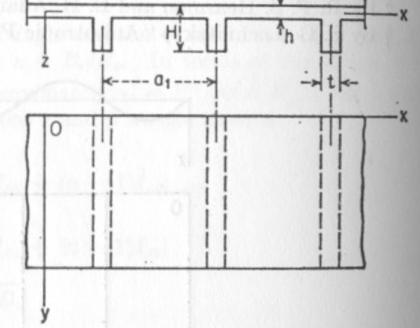

FIG. 188

Plate Cross-stiffened by Two Sets of Equidistant Stiffeners. Provided the reinforcement is still symmetrical about the plating we have

$$D_x = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I_1}{b_1}$$

$$D_y = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I_2}{a_1}$$

$$H = \frac{Eh^3}{12(1-\nu^2)}$$

I_1 being the moment of inertia of one stiffener and b_1 the spacing of the stiffeners in direction x, and I_2 and a_1 being the respective values for the stiffening in direction y.

Slab Reinforced by a Set of Equidistant Ribs. In the case shown in Fig. 188 the theory established in Art. 85 can give only a rough idea of the actual state of stress and

[1] Recommended by Lechnitzky, *op. cit.* For more exact values see N. J. Huffington, *J. Appl. Mechanics*, vol. 23, p. 15, 1956. An experimental determination of the rigidities of stiffened and grooved plates was carried out by W. H. Hoppmann, N. J. Huffington, and L. S. Magness, *J. Appl. Mechanics*, vol. 23, p. 343, 1956.

strain of the slab. Let E be the modulus of the material (for instance, concrete), I the moment of inertia of a T section of width a_1, and $\alpha = h/H$. Then we may assume

$$D_x = \frac{Ea_1h^3}{12(a_1 - t + \alpha^3 t)}$$

$$D_y = \frac{EI}{a_1}$$

$$D_1 = 0$$

The effect of the transverse contraction is neglected in the foregoing formulas. The torsional rigidity, finally, may be calculated by means of the expression

$$D_{xy} = D'_{xy} + \frac{C}{2a_1}$$

in which D'_{xy} is the torsional rigidity of the slab without the ribs and C the torsional rigidity of one rib.[1]

87. Application of the Theory to the Calculation of Gridworks. Equation (213) can also be applied to the gridwork system shown in Fig. 189.

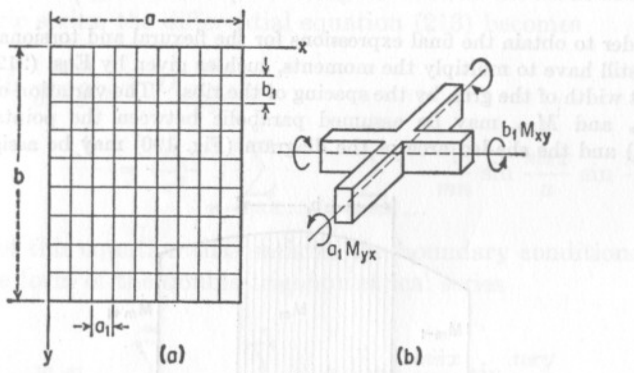

FIG. 189

This consists of two systems of parallel beams spaced equal distances apart in the x and y directions and rigidly connected at their points of intersection. The beams are supported at the ends, and the load is applied normal to the xy plane. If the distances a_1 and b_1 between the beams are small in comparison with the dimensions a and b of the grid, and if the flexural rigidity of each of the beams parallel to the x axis is equal to B_1 and that of each of the beams parallel to y axis is equal to B_2, we can substitute in Eq. (213)

$$D_x = \frac{B_1}{b_1} \qquad D_y = \frac{B_2}{a_1} \tag{a}$$

[1] For a more exact theory concerning slabs with ribs in one or two directions and leading to a differential equation of the eighth order for the deflection see K. Trenks, *Bauingenieur*, vol. 29, p. 372, 1954; see also A. Pflüger, *Ingr.-Arch.*, vol. 16, p. 111, 1947.