

1. We have seen that a number of stencils reduce down to  $y_{n+1} = P(z)y_n$  when applied to  $y' = \lambda y$ , where  $z = \lambda h$ . To leading order, the numerical error obeys  $E_{n+1} = P(z)E_n$  for such stencils. If we wish to avoid exponentially growing errors, we need to enforce the absolute stability criterion  $|P(z)| \leq 1$ . However, it is interesting to note that for some choices of  $\lambda$  the analytic solution of  $y' = \lambda y$  is itself exponentially growing. This motivates us to call a stencil *relatively stable* if the following condition is met:

$$\left| \frac{E_{n+1}}{E_n} \right| \leq \left| \frac{y(x+h)}{y(x)} \right|.$$

On the righthand side,  $y(x)$  and  $y(x+h)$  refer to the analytic solution of  $y' = \lambda y$ . This definition means that a stencil will be relatively stable as long as the numerical errors are growing slower than the actual solution. Plot regions of relative stability in the complex  $z$  plane for the forward and backward Euler methods, the trapezoidal methods, the Huen method, and the classical 4th order Runge-Kutta method.