Radiative Transfer Equation

Let $L_\nu(M,\vec{s})$ be the monochromatic radiance at wavenumber $\nu = 1/\lambda$ (in the interval ν, ν+dv) at a point M in the solid angle $d\omega$ centered on direction $\vec{s}$. $L_\nu(M,\vec{s})$ has dimension $\left[W m^{-2}\, ster^{-1}\, (cm^{-1})^{-1}\right]$.Consider a cylindrical volume element which axis is along s with surface dS and length dl. Let choose an axis along $\vec{s}$ and call $L_\nu(l,\vec{s})$ and $L_\nu(l+dl,\vec{s})$ the radiances at the entrance and exit of the cylinder.

The photons reaching the exit of the cylinder ($L_\nu(l+dl,\vec{s})$) may have three origins (Figure1)

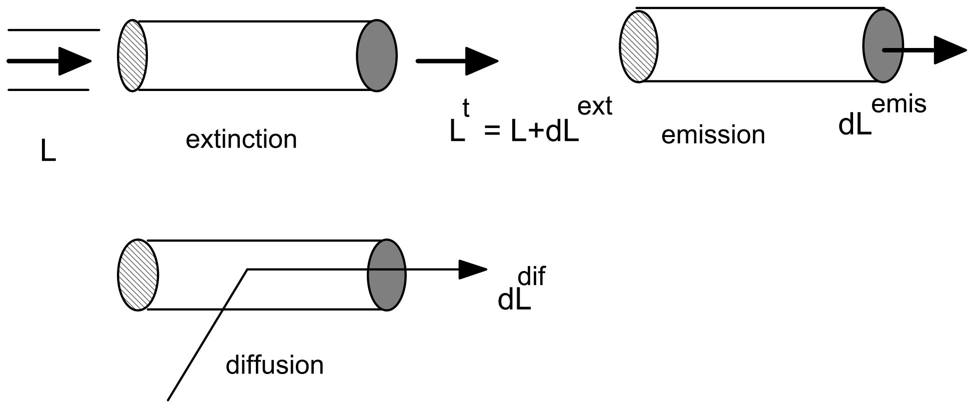

Figure 1

(1) incoming photons, $L_\nu(l,\vec{s})$ of which a fraction, dL_ν^{ext} has been lost either by absorption in the medium or because of a scattering event with a change of their direction. Extinction refers to the total loss due to absorption and scattering and $dL_\nu^{ext} < 0$.
(2) emitted in the volume element
(3) incident in the volume element from direction $s' \neq s$ and scattered in direction s.

The budget writes as

$$L_\nu(l+dl,\vec{s}) = L_\nu(l,\vec{s}) + dL_\nu^{ext} + dL_\nu^{sc} + dL_\nu^{em}$$

The so-called source function is in $\left[Wm^{-2} sr^{-1} (cm^{-1})^{-1} m^{-1}\right]$,

$$J_\nu(l,s) = \frac{dL_\nu^{sc}}{dl} + \frac{dL_\nu^{em}}{dl}$$

and combining (1) and (2) we get the general expression:

$$dL_\nu(l,\vec{s}) = L_\nu(l+dl,\vec{s}) - L_\nu(l,\vec{s}) = dL_\nu^{ext} + J_\nu(l,\vec{s})dl$$

1.1 Extinction

Extinction is caused by two processes: absorption and scattering. Unlike scattering and reflexion, absorption is always associated with a physical change of the medium (temperature, internal energy).

The scattering and absorbing processes are linear and statistically independent, thus

$$dL_\nu^{ext} = -L_\nu(l,\vec{s})\sigma_\nu^{ext} dl$$

$$\sigma_\nu^{ext} = \sigma_\nu^{abs} + \sigma_\nu^{sc}$$

σ_ν^{ext} is the extinction coefficient $[m^{-1}]$.

These coefficients are proportional to the number n of absorbing molecules and/or to the number of scattering particles. One thus defines the efficiency cross sections for one molecule or one particle, s_ν, which has the dimension of a surface $[m^{-2}]$

$$\sigma_\nu = n s_\nu$$

For absorption, the density of the absorbing gas ρ $[kg\,m^{-3}]$ is preferred (mass concentrations),

$$\sigma_\nu^{abs} = \rho s_\nu^{abs}$$

s_ν^{abs} has then dimensions of $[kg\,m^{-2}]^{-1}$

The quantity $du = n\,dl$ or $du = \rho\,dl$ is the total absorber amount along *dl.* For water vapour, the S.I. unit is $[kg\,m^{-2}]$ although $[g\,cm^{-2}]$ or equivalently, precipitable [*cm*] are still commonly $1\,g\,cm^{-2} = 1\,precip.cm$.

For CO2, the usual unit is [*atm-cm*]: 1 *atm-cm* = n_0 = number of molecules of pure CO2 ($n_0 = 6.02\,10^{23}/22400$ = Loschmidt number). At STP (standard temperature and pressure): a path length of *l* [*cm*] with CO2 partial pressure *p* [*atm*] corresponds to *pl* [*atm-cm*].

1.2 Beer-Lambert's law

Assuming that there is neither scattering nor emission in the volume element, then

$$L_\nu(l+dl,\vec{s}) - L_\nu(l,\vec{s}) = dL_\nu^{ext} = -\sigma_\nu^{ext} L_\nu(l,\vec{s})\,dl$$

so that for absorption only,

$$L_\nu(l_1, s) = L_\nu(l_0, s)\,exp\left(-\int_{l_0}^{l_1} \sigma_\nu^{abs}(l')\,dl'\right)$$

1.3 Optical thickness

The absorption optical thickness is defined as

$$\delta_\nu = \int_{l_0}^{l_1} \sigma_\nu^{abs}(l')\,dl' = \int_{l_0}^{l_1} n(l')\,s_\nu^{abs}(l')\,dl'$$

such that

$$L_\nu(l_1, s) = L_\nu(l_0, s)\,e^{-\delta_\nu}$$