
Design of the HCTL-1000's/1100's Digital Filter Parameters by the Combination Method

Application Note 1032

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Introduction

The objective of this application note is to explain the combination method and to show how it can be used for calculation of the HCTL-1000's digital compensation filter parameters to provide a stable, closed loop position control system.

One of the primary advantages of digital closed loop control is that digital compensation can be added to the system to provide the desired closed loop response. Although the HCTL-1000 is a digital controller that employs a discrete sampling rate, the combination method allows the user to model all the system components as continuous (or analog) in the s-plane. This allows the designer to use the experience gained in s-plane, analog control theory to design with the HCTL-1000's digital compensation filter. This application note shows the designer how the combination method can be used for digital compensation design with position control systems that employ a DC motor, an amplifier, an incremental encoder, and the HCTL-1000. The amplifier can be a current or voltage source amplifier. The HCTL-1000 provides an

8 bit motor command port for a linear amplifier and a sign and pulse signal for a pulse width modulated amplifier. Digital compensation filter parameters for two example closed loop systems employing the HCTL-1000 will be designed in appendices A and B of this application note to illustrate the use of the combination method.

The Agilent Technologies HCTL-1000 is a high performance, general purpose, digital motion control IC which can be used in microprocessor based, digital closed loop systems. The HCTL-1000 is designed to communicate with a host microprocessor through an 8 bit parallel I/O port as shown in Figure 1. Commands from the host processor are used by the HCTL-1000 to control an amplifier/motor combination. A motor with an incremental shaft encoder such as a Agilent Technologies HEDS-5500 encoder is connected to the amplifier. The encoder's quadrature output is then fed back into the HCTL-1000 to "close the loop".

Open Loop vs. Closed Loop (Feedback) Control Systems:

A block diagram of an open loop control system is shown in Figure 2. The input to an open loop control system is independent of the output of the system. Therefore, if the open loop system does not provide the desired response to an input, the error between the actual response and the desired response of the system will not be detected. Figure 1 shows the block diagram of a closed loop control system. Closed loop control systems compare their actual output with a desired command input and use the difference

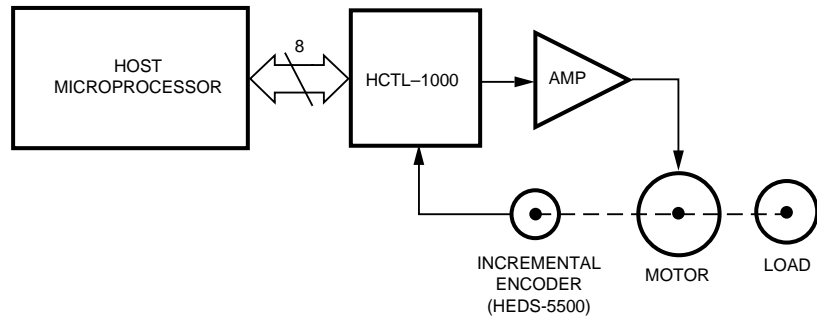


Figure 1. Block Diagram of a Closed Loop Motion Control System with the HCTL-1000.

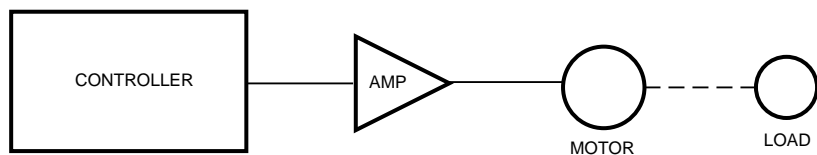


Figure 2. Block Diagram of an Open Loop System.

between them to regulate the system's output. Some of the advantages of closed loop control systems over open loop systems are:

- A. Increased accuracy of the system's response to an input.
- B. Increased system bandwidth (speed). The bandwidth of a system is the range frequencies over which a system will respond "satisfactorily" to an input. The system's bandwidth is a measure of the speed of the system's response. The terms "bandwidth" and "satisfactory response" are explained in more detail in the section of this application note "Bode Plots: Gain Crossover Frequency, Phase Crossover Frequency, Gain Margin, Phase Margin, and System Bandwidth".

The disadvantage of closed loop control systems is that they can become unstable and oscillate. The HCTL-1000 has a built in, programmable digital filter for compensation to increase the stability of the system. In this application note, the combination method will

be used to determine values for the HCTL-1000's digital filter that will make the closed system stable.

Theory

Component Transfer Functions and the Open Loop System Model:

Each of the components in an open or closed loop system can be modeled mathematically by using the "transfer function" of the component. The transfer function models the dynamics of a component by relating the component's output to its input. The transfer functions of commonly used components can be found in most books on control systems.

A block diagram represents the physical system mathematically by showing the transfer functions of each component in its block of the diagram. A typical block diagram with its transfer function is shown in Figure 3. The parameters in each transfer function must be determined to accurately model the system. The response of open loop system is then modeled with a Bode plot to determine the stability and performance of the system when the loop is closed. Based on the characteristics of

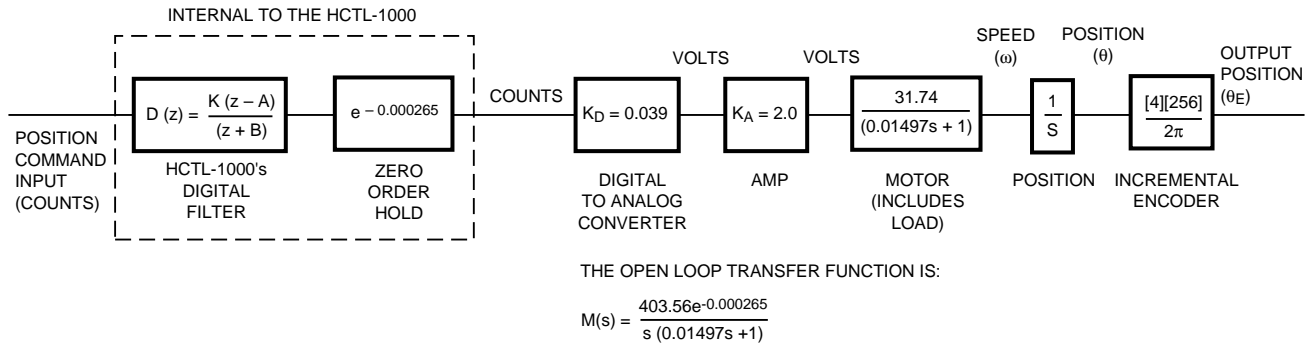


Figure 3. Open Loop Block Diagram of the Example System with a Voltage Source Amplifier.

the open loop system, the HCTL-1000's programmable digital filter parameters A, B, and K can be calculated using the combination method. When the properly calculated values of A, B, and K are programmed into the HCTL-1000, the closed loop system will respond in a stable fashion with the desired performance when the system is run in its linear region of operation. (*i.e.* No component in the system is saturated during the step response.)

This application note assumes that the reader has a general working knowledge of analog control design methods. Specifically, the reader should be familiar with Laplace transforms, the s domain, pole/zero concepts, and Bode plots. Working knowledge of Bode plots is especially important for understanding the combination method for digital compensation filter design. It is for this reason that Bode plot design techniques are reviewed in the next section.

Bode Plots: Gain Crossover Frequency, Phase Crossover Frequency. Gain Margin, Phase Margin, and System Bandwidth:

Bode plots are used to model the frequency characteristics and the relative stability of a system. A

Bode plot consists of two graphs: the magnitude and the phase of a transfer function of a system plotted as a function of frequency. The substitution $s = j\omega$ transforms the s domain transfer function into a complex number whose magnitude and phase can be directly computed at any frequency (ω). The Bode plot provides two values which are used to determine the stability and performance of a closed loop system from plots of the open loop system. These two values are known as the *phase margin* and the *gain margin*.

Figure 4 shows a Bode plot of the transfer function representing the block diagram and transfer function shown in Figure 3. The frequency at which the open loop gain is 1 (0 db) is called the *gain crossover frequency* (ω_C). The frequency where the phase angle is -180 degrees is called the *phase crossover frequency* (ω_P). The *gain margin* (G_M) is defined as the magnitude of the open loop transfer function evaluated at the phase crossover frequency and referenced to the 0 db axis. The *phase margin* (P_{MU}) is defined as the sum of the phase angle of the open loop transfer function evaluated at the gain crossover frequency plus 180 degrees. In cases where both the phase and gain margins of the open

loop transfer function are positive and there are no poles or zeros in the right half of the s-plane, the closed loop system will be stable.

In this application note the phase of the system components and transfer functions will be calculated in radians. When a Bode plot is generated the phase and phase margin will be plotted in degrees. The conversion factor from radians to degrees is:

$$1 \text{ radian} = 57.296 \text{ degrees.} \quad (1)$$

Similarly, the magnitude of the system components and transfer functions will be calculated as decimal quantities (*i.e.* X.XXX) while the magnitude and gain margin will be plotted on a Bode plot in decibels (db). The conversion factor from decimal to db is:

$$\text{db} = 20 \log (\text{decimal}) \quad (2)$$

Use of these conversions is standard practice in motion control design.

The phase margin of a system is a measure of the system's stability. The higher the phase margin of the open loop system, the more stable the closed loop system becomes. The designer selects a compensated phase margin (P_{HC}) to provide the desired closed loop

system response to a step input and then designs a compensator (digital filter) to provide the desired phase margin. A compensated phase margin of 30 to 60 degrees is generally recommended for stable designs. Figure 5 shows the effect of different compensated phase margins on the step response of an actual closed loop system employing a small DC motor, and a voltage source amplifier.

Increasing the gain margin of the open loop system also increases the stability of the closed loop system. The combination method does not directly assist the designer in easily computing a new gain margin for the compensated system.

The system bandwidth is the range of frequencies over which a system will respond "satisfactorily" to an input. "Satisfactory response" of the system is defined as the range of frequencies over which the magnitude of the Bode plot does not differ from its DC value by more than -3db.

In this application note, the closed loop bandwidth (BW) will be approximated as equal to the open loop gain crossover frequency (ω_C) for systems with a high phase margin:*

$$\omega_C \cong BW \quad (3)$$

For systems with a low phase margin, the bandwidth (BW) is greater than or equal to the gain crossover frequency (ω_C) of the system and a corresponding increase in overshoot occurs;*

$$BW \geq \omega_C \quad (4)$$

*Note: These approximations are based on a two pole dominant closed loop system model.

For more information on approximating the closed loop bandwidth by the desired open loop gain crossover frequency and other relationships in these systems involving bandwidth, gain crossover frequency, risetime, falltime, percent overshoot, etc. several references on motion control are listed in the bibliography at the end of this application note.

Summary of the Design Procedure

The HCTL-1000's Digital Filter Design Procedure Using the Combination Method:

The procedure recommended in this application note for designing closed loop systems that employ the HCTL-1000's digital filter is as follows:

- A. Choose a motor to drive the required load. One common criteria for choosing a

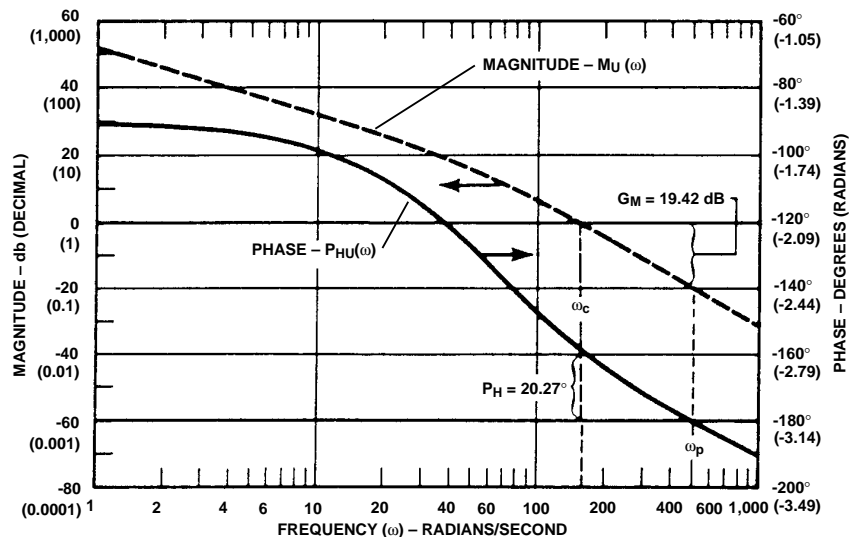


Figure 4. Bode Plot for the Open Loop System and Transfer Function Shown in Figure 3.

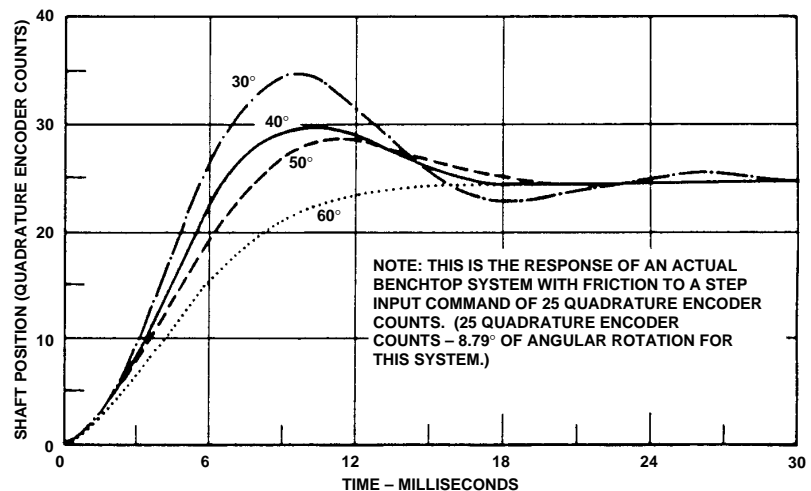


Figure 5. Shaft Position vs. Time for a Selection of Phase Margins.

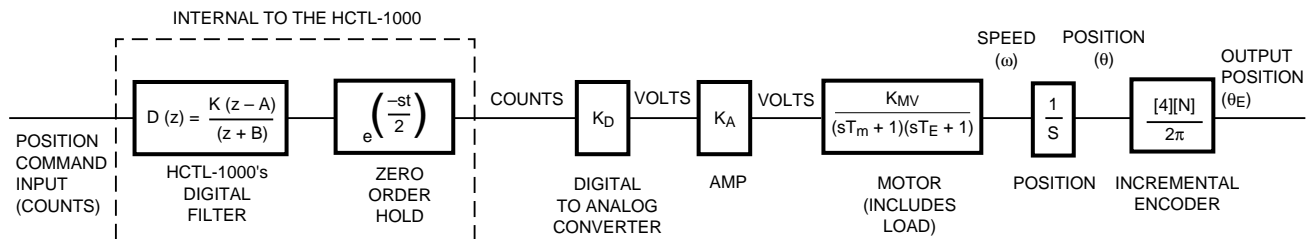
- motor is based on the ratio of the torque developed by the motor to the moment of inertia of the motor's armature and the load. It is desirable to choose a motor with a low armature inertia that can develop a high torque to quickly accelerate and decelerate the load (i.e. the motor has a high torque constant $[K_T]$)
- B. Choose a digital incremental encoder with quadrature output to monitor the position of the motor's shaft based on the encoder resolution and accuracy required for the application.
- C. Choose an amplifier to drive the motor. The amplifier must be capable of supplying the current and voltage required by the motor for the load conditions.
- D. Model the open loop transfer function of the system using s-plane transfer functions. A Bode plot showing the phase margin and gain margin of the open loop system can then be drawn from the open loop transfer function.
- E. Choose the desired closed loop response (step response and bandwidth). The desired phase margin and gain crossover frequency for the compensated system can then be determined from the desired closed loop response.
- F. Design the HCTL-1000's digital compensation filter based on the desired phase margin and gain crossover frequency for the compensated system.

System Modeling

Modeling the System Components in the S-Plane: The Open Loop Transfer Function

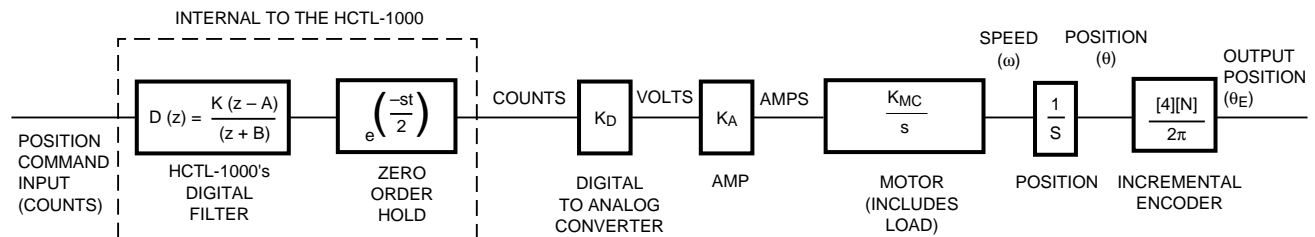
Introduction to the open loop transfer function:

Figures 6A and 6B show open loop block diagrams for systems which include a controller (the HCTL-1000's digital filter and the zero-order-hold representing the HCTL-1000's digital sampling delay), an amplifier, a motor, a load, and an incremental encoder. The block diagram in Figure 6A shows an open loop motion control system that employs a voltage source amplifier. The block diagram includes both the mechanical time constant of the motor and the electrical time constant of the motor. The block diagram shown in Figure 6B is used for systems that employ a current source



THE OPEN LOOP TRANSFER FUNCTION IS: $M(s) = \left[e^{\left(\frac{-st}{2}\right)} \right] \left[K_D \right] \left[K_A \right] \left[\frac{K_{MV}}{(sT_m + 1)(sT_E + 1)} \right] \left[\frac{1}{s} \right] \left[\frac{[4][N]}{2\pi} \right]$

Figure 6a. The Open Loop Transfer Function of a System with a Voltage Source Amplifier. Note that the motor model includes the mechanical time constant (T_m) and the electrical time constant (T_E) of the motor.



THE OPEN LOOP TRANSFER FUNCTION IS: $M(s) = \left[e^{\left(\frac{-st}{2}\right)} \right] \left[K_D \right] \left[K_A \right] \left[\frac{K_{MC}}{s} \right] \left[\frac{1}{s} \right] \left[\frac{[4][N]}{2\pi} \right]$

Figure 6b. The Open Loop Transfer Function of a System with a Current Source Amplifier.

amplifier. This model does not use the electromechanical time constants of the motor.

The open loop transfer function $M(s)$ for these systems is determined by multiplying all of the individual component transfer functions together.

$$M(s) = [\text{DAC gain (if used)}] [\text{Zero order hold}] [\text{Amp gain}] [\text{Motor transfer function}] [\text{encoder quadrature counts}/(2\pi)] \quad (5)$$

The following sections describe the method for determining the transfer function of each component, determining the magnitude and phase contribution of each component, computing the open loop transfer function of the system, and then drawing a Bode plot of the open loop system.

The Zero Order Hold (ZOH) transfer function:

The zero order hold (ZOH) models the delay of the discrete sampling time of the HCTL-1000. The transfer function ($Z(s)$) of the zero order hold (ZOH) is:

$$Z(s) = e^{-st/2} \quad (6)$$

Where:

t = the sampling time (ie. time between successive samples) of the HCTL-1000 (in seconds).

This transfer function of the ZOH allows the designer to model the time delay of the HCTL-1000 as a continuous element in the system. Analog, s-plane design techniques can therefore be used to analyze the system.

The ZOH's contribution to the phase ($P_{ZOH}(\omega)$) of the open loop transfer function is:

$$P_{ZOH}(\omega) = -\frac{[\omega] [t]}{2} \text{ (radians)} \quad (7)$$

Where:

ω = the frequency of interest (in radians/sec)

t = the sampling time of the HCTL-1000 (in seconds)

Phase lag is added to the system by increasing the time delay between successive samples of the ZOH. It is usually desirable therefore to choose the fastest sampling time possible so as to induce the least amount of phase lag. Generally, the sampling frequency ($1/t$) should exceed the system bandwidth (in hertz) at least tenfold:

$$\text{Sampling time } (t) < [1/10 \text{ System bandwidth}] \quad (8)$$

The ZOH provides unity gain at all frequencies of the system.

The sampling time (t), can be programmed into the HCTL-1000 through register ROFH. The sampling time (t) can be determined from the equation:

$$t = [16] [\text{ROFH} + 1] [1/f_{\text{CLK}}] \text{ (seconds)} \quad (9)$$

where:

t = sample time (in seconds)

f_{CLK} = freq- of the HCTL-1000's external clock (1 or 2 MHz)

ROFH = sample timer register

The limits of minimum values (ie. fastest sampling times) that can be programmed into register ROFH are:

Mode:	ROFH Contents Minimum Limit:
Position Control	7 (decimal)
Proportional Velocity Control	7 (decimal)
Trapezoidal Profile Control	15 (decimal)
Integral Velocity Control	15 (decimal)

The Digital to Analog Converter (DAC) transfer function:

Linear voltage source and current source amplifiers require a digital to analog converter to interface with the HCTL-1000.

The DAC's transfer function is simply its gain (K_D). The digital to analog converter (DAC) transfer function can include an electrical time constant, but it is usually neglected because the bandwidth of the DAC is generally much higher than the desired bandwidth of the closed loop system.

The gain (K_D) is the DAC's contribution to the magnitude of the open loop transfer function. The formula for computing the gain (K_D) of the DAC is:

Gain (K_D) =

$$\frac{\text{Voltage range (volts)}}{2^N \text{ (bits or counts)}} \quad (10)$$

Example:

For an eight bit DAC with an output range of -5 to +5 volts the equation would be:

$$K_D = \frac{5 - (-5) \text{ volts}}{2^8 \text{ counts}} = \frac{10}{256} =$$

.039 volts/count

Note that the units of the DAC gain is volts/count. This gives the combination of the linear voltage

source amplifier and the DAC the units of volts/count, and the linear current source amplifier/DAC the units of amps/count.

The DAC does not contribute a phase shift to the open loop transfer function.

The amplifier transfer function: The amplifier transfer function is its gain (K_A). The choice between using either a current or voltage amplifier affects the *motor* transfer function. The amplifier's transfer function can include an electrical time constant, but this is usually neglected as the amplifier's bandwidth is generally much higher than the desired bandwidth of the closed loop system. In general, if the desired bandwidth of the system is 10 times smaller than the amplifier's bandwidth, then the amplifier's electrical time constant can be neglected. Note that the units of the gain (K_A) vary with the type of amplifier chosen.

There are four types of amplifiers that can be used with the HCTL-1000:

- Voltage source – pulse width modulated
- Current source – pulse width modulated
- Voltage source – linear
- Current source – linear

The HCTL-1000 provides an 8 bit binary word on its motor command port to drive a linear amplifier, and a pulse and sign signal to drive a pulse width modulated amplifier.

The amplifier does not contribute a phase shift to the open loop transfer function when its electrical time constant is neglected. The gain (K_A) is the amplifier's contribution to the magnitude of the open loop transfer function.

The gain and bandwidth of most linear amplifiers can be obtained from the manufacturer's data sheet. If the user desires to measure the amplifier's gain, the DC-output for a given DC-input can be measured directly. The units for the gain (K_A) of a linear voltage source amplifier are volts/volt. The units for the gain (K_A) of a linear current source amplifier are amps/volt.

A simplified formula that will be used in this application note for computing the gain of a pulse width modulated (PWM) amplifier is:

$$K_A = \frac{\left[\begin{array}{c} \text{Maximum} \\ \text{signal} \\ \text{output} \end{array} \right] - \left[\begin{array}{c} \text{Minimum} \\ \text{signal} \\ \text{output} \end{array} \right] \text{ (current or voltage)}}{\left[\begin{array}{c} \text{duty cycle} \\ \text{input to} \\ \text{get maximum} \\ \text{output} \end{array} \right] - \left[\begin{array}{c} \text{duty cycle} \\ \text{input to} \\ \text{get minimum} \\ \text{output} \end{array} \right]} \quad (11)$$

The gain of a PWM amplifier is measured by inputting a waveform with a known duty cycle. If a voltage source PWM amplifier is being measured, the output voltage can be read with a DC voltmeter to average the output of the amplifier. If a current source PWM amplifier is being measured, the current can be dropped across a load of known value to produce a voltage and then that voltage can be read with a DC voltmeter to average the output of the amplifier.

For the HCTL-1000, the duty cycle of the input to the PWM amplifier is regulated by register R09H. Register R09H has a range of 64H to 9CH (-100 to +100 decimal). The units of the gain (K_A) are for the HCTL-1000 and PWM amplifier combination

are therefore volts or current per count.

The DC motor transfer function:

The DC motor transfer function is calculated based on the total inertia of the system, the parameters used to describe the motor's dynamic response, and the type of amplifier used to drive the motor. Different transfer functions are used for a motor driven by a voltage source amplifier than for a motor driven by a current source amplifier. The total inertia of the system and the parameters used to describe the system's dynamic response are the same for both motor transfer functions. They are described in the next two sections. Following these two sections are two sections which describe the transfer functions for motors driven by current source amplifiers and motors driven by voltage source amplifiers.

1. DC motor manufacturer's motor parameters:

The DC motor parameters of interest that describe the electro-mechanical characteristics of motor mathematically are: the torque constant (K_T) [in N-m/amp], the moment of inertia of the armature (J_M [in kg-m²], the terminal resistance (R) [in ohms], the voltage constant (K_E) [in volt-sec/rad], and the armature inductance (L) [in henries]. These motor parameters can all be obtained directly from the motor manufacturer's data sheet.

2. Total system moment of inertia:

The first step in determining the moment of inertia of the system is to determine the moment of inertia of the load (J_L). For many loads the moment of inertia of

the load (J_L) can be read directly from the manufacturer's data sheet. The moment of inertia of the incremental encoder's codewheel (J_C) can also be read directly from the encoder manufacturer's data sheet. It is recommended that a book on mechanical design be consulted to find the moment of inertia of other loads.

The total moment of inertia for the system (J) includes the motor's armature inertia (J_M), the load inertia (J_L), and the encoder codewheel inertia (J_C). Therefore:

$$J = J_M + J_L + J_C \quad (12)$$

3. DC motors driven by voltage source amplifiers:

The DC motor transfer function for a motor driven by a voltage source amplifier is:

$$G(s) = \frac{\text{position output}}{\text{voltage input}} = \frac{\theta(s)}{v(s)} \\ = \frac{K_{MV}}{s(sT_M + 1)(sT_E + 1)} \quad (13)$$

The term s in the denominator indicates integration due to the fact that position is the output. Note that the transfer function for a motor driven by a voltage source amplifier has three main components; a gain constant (K_{MV}), a mechanical time constant (T_M) [in seconds] and an electrical time constant (T_E) [in seconds]. The gain constant (K_{MV}) of the motor driven by a voltage source amplifier is defined as the velocity at which the motor's shaft will turn when 1 volt is applied to the input terminals. The mechanical and electrical time constants of the motor describe the response time of the motor to an input. The mechanical time constant (T_M) of

a motor is defined as the time required for the unloaded motor to reach 63.2% of its final velocity after the application of a DC step voltage to the motor's armature. The electrical time constant (T_E) of a motor is defined as the ratio of the armature inductance to the armature resistance.

The constants in the motor transfer functions T_E , T_M , and K_{MV} are determined by the manufacturer's motor parameters and the total system inertial load that the motor is driving.

The equations for the mechanical (T_M) and electrical (T_E) time constants are:

$$T_M = \frac{[R] [J]}{[K_E] [K_T]} \quad (\text{seconds}) \quad (14)$$

$$T_E = \frac{L}{R} \quad (\text{seconds}) \quad (15)$$

The equation to calculate the gain constant (K_{MV}) is:

$$K_{MV} = \frac{1}{K_E} \quad (\text{rad/volt-sec}) \quad (16)$$

Where:

R = the terminal resistance of the motor (in ohms)

J = the total system moment of inertia (in kg-m²)

K_E = the voltage constant of the motor (volt-sec/rad)

K_T = the torque constant of the motor (in N-m/amp)

The electrical time constant can be neglected if the mechanical time constant is greater than 10 times the value of the electrical time constant. This reduces the motor transfer function to:

$$G(s) = \frac{\text{position output}}{\text{voltage input}} = \frac{\theta(s)}{v(s)}$$

$$= \frac{K_{MV}}{s(sT_M + 1)} \quad (17)$$

The phase contribution of a motor [$P_M(\omega)$], driven at frequency (ω) by a voltage source amplifier is:

$$P_M(\omega) = -\arctan([\omega][T_M]) - \\ -\arctan([\omega][T_E]) - \\ \left[\frac{\pi}{2} \right] \quad (\text{radians})^* \quad (18)$$

The magnitude contribution of a motor [$M_M(\omega)$], driven at frequency (ω) by a voltage source amplifier is:

$$M_M(\omega) = \frac{K_{MV}}{\omega \sqrt{1 + ([\omega][T_M])^2} \sqrt{1 + ([\omega][T_E])^2}} \quad (19)$$

Where:

ω = the frequency of interest (in radians)

T_M = the mechanical time constant of the motor (in seconds)

T_E = the electrical time constant of the motor (in seconds)

K_{MV} = the gain constant of the motor (in rad/volt-sec)

Note: For cases where the electrical time constant has been neglected let $T_E = 0$.

4. DC motors driven by current source amplifiers:

The DC motor transfer function for a motor driven by a current source is:

$$G(s) = \frac{\text{position output}}{\text{current input}} = \frac{\theta(s)}{1(s)} \\ = \frac{K_{MC}}{s^2} \quad (20)$$

The gain constant (K_{MC}) for systems employing a current source amplifier is computed as:

$$K_{MC} = \frac{K_T}{J} \text{ (rad/amp sec}^2\text{)} \quad (21)$$

Where:

J = the total system moment of inertia (in kg-m²)

K_T = the torque constant of the motor (in N-m/amp)

The phase contribution of a motor [$P_M(\omega)$] driven at frequency (ω) by a current source amplifier is:

$$P_M(\omega) = -\pi \text{ (radians)} \quad (22)$$

The magnitude contribution of a motor [$M_M(\omega)$], driven at frequency (ω) by a current source amplifier is:

$$M_M(\omega) = \frac{K_{MC}}{(\omega)^2} \quad (23)$$

Where:

ω = the frequency of interest (in radians)

K_{MC} = the gain constant of the motor (in rad/amp-sec²)

The encoder transfer function:

The incremental encoder's transfer function (E) is:

$$E = \frac{C}{2\pi} = \frac{[4] [N]}{2\pi} \text{ (count/rad)} \quad (24)$$

Where:

C = quadrature counts per revolution of the encoder's codewheel

N = the number of slits in the encoder's codewheel

The 2π indicates the number of radians in one revolution of the codewheel. Because the HCTL-1000 utilizes a four times quadrature decoder, the closed loop system has four times the resolution (C) as the number of slits in the codewheel (N). The number of slits in the codewheel (N) can be found from the encoder manufacturer's data sheet.

The encoder's codewheel count does not contribute to the phase of the open loop transfer function. The magnitude contribution of the encoder's codewheel count (E) to the open loop transfer function is independent of frequency:

$$E = \frac{C}{2\pi} \text{ (count/rad)} \quad (25)$$

Where:

C = quadrature counts per revolution of the encoder's codewheel

The phase and magnitude contribution of the codewheel's inertia (J_C), has already been considered in the total system load on the motor.

Determination of the Open Loop transfer function:

Now that all of the individual transfer functions have been determined for each component of the open loop system, the open loop transfer function $M(s)$ can be determined by multiplying the individual transfer functions together.

For example, the open loop transfer function for the system with the current source amplifier shown in Figure 6B is:

$$M(s) = [e^{-st/2}][K_d][K_a]$$

$$\left[\frac{K_M}{s} \right] \left[\frac{1}{s} \right] \left[\frac{C}{(2\pi)} \right] \quad (26)$$

Determination of the gain crossover frequency, phase crossover frequency, gain margin, and phase margin of the open loop transfer function using a Bode plot:

A Bode plot can be created using the open loop transfer function generated in the last section.

The purpose of creating the Bode plot is to show the characteristics of the uncompensated system over frequency. The Bode plot also shows the designer how much compensation should be added to the system to achieve the desired system bandwidth and stability.

The method for creating a Bode plot from the open loop transfer function is to plot the phase and magnitude of the open loop transfer function over frequency. In the following equations for the phase and magnitude the constants are:

E = the gain of the incremental encoder (counts/ radian)

K_A = the gain of the amplifier (volts/volt for linear voltage source amplifiers, amps/volt for linear current source amplifiers, volts/count for PWM voltage source amplifiers, amps/count for PWM current source amplifiers.)

K_D = the gain of the DAC (volts/count)

K_{MC} = the gain constant of a motor driven by a current source amplifier (rad/amp-sec²)

K_{MV} = the gain constant of a motor driven by a voltage source amplifier (rad/volt-sec)

$M_M(\omega)$ = the magnitude of the motor transfer function

$P_M(\omega)$ = the phase of the motor transfer function (radians)

$P_{ZOH}(\omega)$ = the phase of the zero order hold transfer function (radians)

t = the sampling time of the HCTL-1000 (seconds)

T_E = the electrical time constant of the motor (seconds)

T_M = the mechanical time constant of the motor (seconds)

ω = the frequency of interest (radians)

The phase $[P_{HU}(\omega)]$ of the uncompensated open loop transfer function over frequency (ω) is determined by the equation:

$$P_{HU}(\omega) = P_M(\omega) + P_{ZOH}(\omega) \text{ (radians)} \quad (27)$$

For a system employing a voltage source amplifier, the following equation is used for calculation of the phase $[P_{HU}(\omega)]$ of the open loop transfer function:

$$P_{HU}(\omega) = -\arctan([\omega][T_M]) - \arctan([\omega][T_e]) - \left[\frac{\pi}{2} \right] - \left[\frac{[\omega][t]}{2} \right] \text{ (rad)}^* \quad (28)$$

For systems with a voltage source amplifier where the electrical time constant (T_E) has been neglected, the equation becomes:

$$P_{HU}(\omega) = -\arctan([\omega][T_M]) - \left[\frac{\pi}{2} \right] - \left[\frac{[\omega][t]}{2} \right] \text{ (rad)}^* \quad (29)$$

For systems with a current source amplifier the equation becomes:

$$P_{HU}(\omega) = -\pi - \left[\frac{[\omega][t]}{2} \right] \text{ (rad)}^* \quad (30)$$

*Note: Multiply $P_{HU}(\omega)$ by 57.296 (deg/rad) to obtain the phase in degrees. (31)

The magnitude $[M_U(\omega)]$ of the uncompensated open loop transfer function over frequency (ω) is:

$$M_U(\omega) = [M_M(\omega)][K_D][K_A][E] \quad (32)$$

The magnitude $[M_U(\omega)]$ of the uncompensated open loop transfer function of a system with a voltage amplifier is:

$$M_U(\omega) = \frac{[K_{MV}][K_D][K_A][C/2\pi]^{**}}{\omega \sqrt{[1 + ([\omega][T_M])^2]} \sqrt{[1 + ([\omega][T_e])^2]}} \quad (33)$$

For systems with a voltage source amplifier where the electrical time constant (T_E) has been neglected, the uncompensated magnitude $[M_U(\omega)]$ is:

$$M_U(\omega) = \frac{[K_{MV}][K_D][K_A][C/2\pi]^{**}}{\omega \sqrt{[1 + ([\omega][T_M])^2]}} \quad (34)$$

The uncompensated magnitude $[M_U(\omega)]$ for the loop transfer function of systems with a current source amplifier is:

$$M_U(\omega) = \frac{[K_{MC}][K_D][K_A][C/2\pi]^{**}}{(\omega)^2} \quad (35)$$

**Note: To express the magnitude in db, use:

$$\text{db} = 20 \log [M_U(\omega)] \quad (36)$$

When the equations for the phase $[P_{HU}(\omega)]$ and gain $[M_U(\omega)]$ of the open loop transfer function have been determined in terms of component parameters and frequency (ω), then different values of frequency (ω) may be substituted into these equations to create a Bode plot of the phase and gain of the open loop system.

The next step is to evaluate the phase and gain crossover frequencies and the phase and gain margins from the Bode plot.

To find the phase crossover frequency (ω_p) let the phase $[P_{HU}(\omega)]$ equal $-\pi$ radians (i.e. -180 degrees) in the appropriate phase equation and solve for the frequency (ω). This value of the frequency (ω) is the phase crossover frequency (ω_p).

(37)

To find the gain crossover frequency (ω_c) let the magnitude $[M_U(\omega)]$ equal 1 (0 db) in the appropriate magnitude equation and solve for the frequency (ω). This value of the frequency (ω) is the gain crossover frequency (ω_c).

(38)

The gain margin (G_M) [in db] is:

$$G_M = \frac{1}{M_U(\omega_p)} \text{ (decimal)} = 20 \log \left[\frac{1}{M_U(\omega_p)} \right] \text{ (db)} \quad (39)$$

$$G_M = -20 \log M_U(\omega_p) \quad (40)$$

Where:

$M_U(\omega_p)$ = the magnitude of the open loop system evaluated at the phase crossover frequency (ω_p).

The phase margin (P_H) is

$$(P_H) = [180 \text{ deg} + (57.296 \text{ (deg/rad)} \times P_{HU}(\omega_c))] \text{ (degrees)} \quad (41)$$

Where:

$P_{HU}(\omega_c)$ = the phase of the open loop system evaluated at the gain crossover frequency (ω_c).

Figure 4 shows an example of a Bode plot drawn from the open loop transfer function of the block diagram shown in Figure 3. The Bode plot includes the phase crossover frequency, gain crossover frequency, phase margin, and

gain margin of the uncompensated, open loop system.

Once the Bode plot of the uncompensated, open loop transfer function has been completed, the designer can determine how much compensation should be added to the system to achieve the desired system bandwidth and stability. A method for determining how much compensation to add to the system is described in the next section.

Modification of the Open Loop System Transfer Function Using Compensation to Achieve the Desired Closed Loop Characteristics

Once the phase and gain margins of the open loop transfer function are known at the original gain and phase crossover frequencies, a new phase margin and gain crossover frequency can be chosen to achieve a stable closed loop system at a bandwidth equal to or higher than the uncompensated bandwidth.

The open loop transfer function can be modified with the HCTL-1000's digital compensation filter to improve the bandwidth (speed) and stability of the system when the loop is closed with the HCTL-1000. The stability of the closed loop system can be increased by increasing the phase margin of the system. The bandwidth and the gain crossover frequency of the system can be increased by increasing the gain of the system.

Generation of new desired open loop characteristics using the desired closed loop bandwidth:

In this application note, the desired closed loop bandwidth

(BW) will be *approximated as equal* to the desired open loop gain crossover frequency (ω_C') for high phase margin systems.

$$\omega_C' \equiv \text{BW [closed loop]} \quad (42)$$

Based on the required step response of the system, the user should choose a realistic closed loop bandwidth (BW), and compensated phase margin (P_{HC}). The digital compensation filter can contribute a maximum phase lead of approximately 80 degrees to the uncompensated phase margin. The amount of bandwidth that can be added by the digital filter depends on the type of system being compensated. Both the gain of the amplifier (K_A) and the gain of the digital filter (K) can be used to change the gain margin of the system.

The desired gain crossover frequency (ω_C') is now the frequency of interest that will be substituted into *both* the phase and gain equations described in the next section.

Determination of the phase margin and gain of the open loop transfer function at the desired, open loop gain crossover frequency:

The next step is to calculate the phase margin (P_{MU}) of the uncompensated system at the desired frequency (ω_C') and the gain (K_F) required to achieve the desired frequency (ω_C').

The uncompensated phase margin (P_{MU}) at the desired open loop gain crossover frequency (ω_C') can be calculated by substituting ω_C' as the frequency of interest (ω) in the appropriate equation for the phase [$P_{HU}(\omega)$]:

$$P_{MU} = [180 \text{ deg} + [P_{HU}(\omega_C')]$$

$$\times 57.296(\text{deg/rad})] \text{ degrees} \quad (43)$$

Where:

$P_{HU}(\omega_C')$ = the phase angle of the open loop system evaluated at the desired gain crossover frequency (ω_C').

The phase lead (P_L) that the HCTL-1000's digital filter must provide to the closed loop system to make it stable can be found from the following equation:

$$P_L = P_{HC} - P_{MU} \text{ (degrees)} \quad (44)$$

Where:

P_{HC} = the desired compensated margin at ω_C' (degrees)

P_{MU} = the uncompensated phase margin at ω_C' (degrees)

The gain (K_F) required from the digital compensation filter to make the overall gain equal to one at the desired gain crossover frequency (ω_C') is:

$$K_F = \frac{1}{M_U(\omega_C')} \quad (45)$$

Where:

$M_U(\omega_C')$ = the magnitude of the open loop system evaluated at the desired gain crossover frequency (ω_C').

The HCTL-1000's digital compensation filter, and a method for computing the values for the gain factor (K), the pole (B) and the zero (A) are discussed in the next section of this application note.

Use of the Combination Method to Design the HCTL-1000's Digital Compensation Filter: Characteristics of the HCTL-1000's digital filter:

The digital compensation filter of the HCTL-1000 is of the form:

$$D(z) = \frac{[K][z - A]}{[4][z + B]} = \left[\frac{K}{4} \right] \left[\frac{z - A}{z} \right] \left[\frac{z}{z - B} \right] \quad (46)$$

Both the pole term (B) and the zero term (A) contribute phase lead to the system. The K term is an independent gain factor that can be used to raise the system gain for higher bandwidth, and to compensate for the gain reduction associated with the digital filter's pole and zero term.

The combination method and the phase and magnitude graphs:

The combination method is a graphical analysis technique that can be used to design HCTL-1000's digital compensator filter. The four graphs shown in figures 7, 8, 9, and 10 are employed by the combination method to show the phase of digital filter's pole [$P_P(\omega_N)$], the phase of digital filter's zero [$P_Z(\omega_N)$], the magnitude of the digital filter's pole [$M_P(\omega_N)$], and the magnitude of the digital filter's zero [$M_Z(\omega_N)$]. Note that the magnitude and phase for both the pole and zero terms are plotted on individual graphs. The combination method employs these plots to determine values for the pole (B), zero (A), and gain (K) that will provide the required phase lead and gain for the desired closed loop system response to a step input over a specified bandwidth.

Normalized frequencies are used in these graphs to allow digital compensation to be designed for systems with a wide range of bandwidths and sample times. The normalized frequency [$\omega_N(\omega)$] is calculated as the frequency (ω) multiplied by the sampling time (t) of the system:

$$\omega_N(\omega) = [\omega] [t] \text{ (radians)} \quad (47)$$

ω_{NC}' is the desired gain crossover frequency (ω_C'), normalized to the sampling time (t) of the HCTL-1000 by equation (47):

$$\omega_{NC}' = [\omega_C] [t] \text{ (radians)} \quad (48)$$

Where:

$$\begin{aligned} \omega_C' &= \text{the desired gain crossover frequency (radians/sec)} \\ t &= \text{the sampling time of the HCTL-1000 (seconds)} \end{aligned}$$

The normalized desired gain crossover frequency (ω_{NC}') is used as the frequency at which the phase and magnitude contributions of the digital filter's pole and zero term are evaluated.

Figures 11, 12, 13, and 14 show expanded versions of the phase and magnitude graphs for the pole (B) and zero (A) term. These graphs have been expanded for values of the normalized frequency [$\omega_N(\omega)$] between 0.0 and 0.65. This range of values is most often used due to the requirement that the sampling frequency (1/t) must be at least 10 times the system bandwidth (in Hertz).

$$\begin{aligned} t &\leq 1/(10 \times BW) \\ t &\leq (2 \times \pi)/(10 \times \omega_C') \end{aligned} \quad (8)$$

Now:

$$\omega_{NC}' \leq \omega_C' \times t \quad (48)$$

$$\begin{aligned} \omega_{NC}' &\leq \omega_C' \times (2 \times \pi)/(10 \times \omega_C') \\ \omega_{NC}' &\leq (2 \times \pi)/10 \end{aligned}$$

Therefore:

$$\omega_{NC}' \leq .628$$

Note that for all four graphs the value of the pole term (B) and the zero term (A) is always less than or equal to 1.0 for proper system design. Also note that when the phase lead from the HCTL-1000's compensator is increased the gain of the system is decreased by the pole and zero terms.

The normalized frequency plots for the phase and magnitude of the pole and zero terms allow the designer to use analog design techniques to determine values for the HCTL-1000's digital compensation filter. This is accomplished by mapping from the digital z-plane to the analog s-plane through the equation $z = e^{st} = e^{j\omega t}$ where t is the sampling time of the system, and $\omega_N(\omega)$ is the normalized frequency of the system.

Generation of the phase and magnitude graphs:

The following text describes the method and equations for generating the phase and magnitude graphs. This information is useful for designers who would like to implement the combination method in software, but it is not necessary to read this text for the designer to use the graphs for filter design.

The graphs for the pole term are generated from the $z/(z + B)$ portion of equation (46). The plot for the phase of the pole term is derived by substituting $z = e^{j\omega t}$ into $z/(z + B)$ and then taking the argument of this term to get:

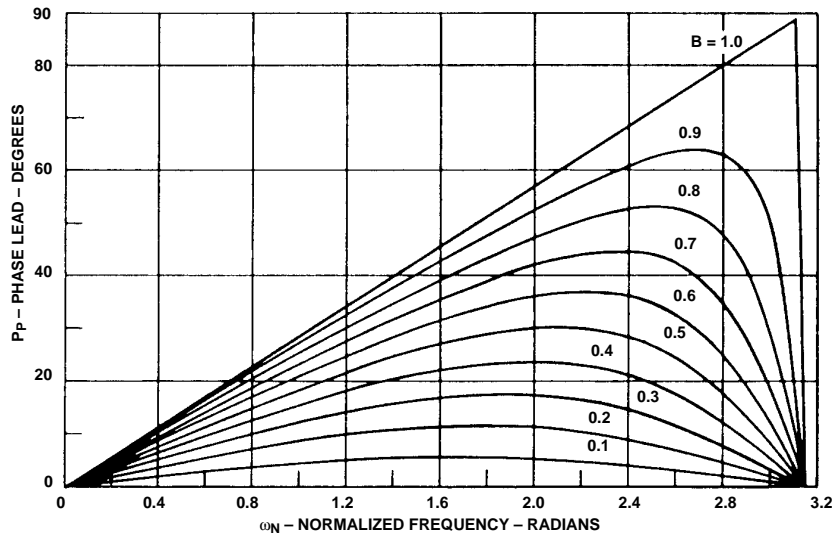


Figure 7. Phase Lead Contribution of the Pole Term.

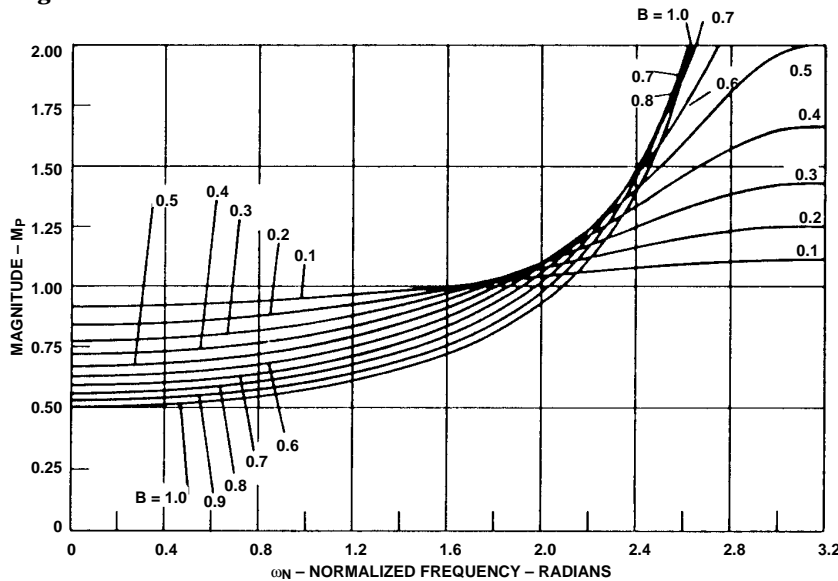


Figure 8. Magnitude Contribution of the Pole Term.

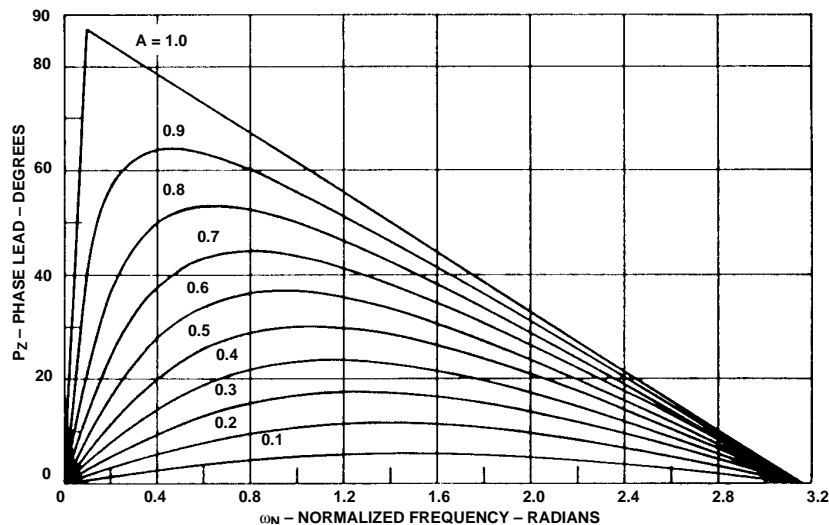


Figure 9. Phase Lead Contribution of the Zero Term.

$$P_P(\omega_N) = \arg \left[\frac{e^{j\omega t}}{e^{j\omega t} + B} \right] = \arctan \left[\frac{B \sin \omega t}{(B \cos \omega t) + 1} \right] \quad (49)$$

The magnitude of the pole term is derived from:

$$M_P(\omega_N) = \left[\frac{e^{j\omega t}}{e^{j\omega t} + B} \right] = \frac{1}{\sqrt{[(B \cos \omega_T) + 1]^2 + [B \sin \omega_T]^2}} \quad (50)$$

The graphs of the zero term are derived from the $(z-A)/z$ portion of equation (46). The plot of the phase of the zero term is found by substituting $z = e^{j\omega t}$ into $(z-A)/z$ and then taking the argument of this term to get:

$$P_Z(\omega_N) = \arg \left[\frac{e^{j\omega t} - A}{e^{j\omega t}} \right] = \arctan \left[\frac{A \sin \omega t}{(-A \cos \omega t) + 1} \right] \quad (51)$$

The plot of the magnitude of the zero term is derived from:

$$M_Z(\omega_N) = \left[\frac{e^{j\omega t} - A}{e^{j\omega t}} \right] = \frac{1}{\sqrt{[(-A \cos \omega t) + 1]^2 + [A \sin \omega t]^2}} \quad (52)$$

Determination of the HCTL-1000's Digital Filter Parameters: A, B, and K by the Combination Method:

Three programmable components, the zero (A), the pole (B), and the gain (K) are accessible to the user through registers 20H, 21H, and 22H of the HCTL-1000 respectively. By changing these three numbers, the user can change the response of the system to an input. Table 1

Table 1: Effect of changing digital filter parameters A, B, or K on the response of the system to a step input.

Increase in Parameter	Stability	Effect on Response	Stiffness
A	Better	Faster	Decreases
B	Slightly Better	Faster	Decreases
K	Worse	Faster	Increases

shows the effect of changing each of these parameters on the response of the system to a position step input. The combination method to find values for the pole (B), zero (A), and gain (K) is as follows:

1. On Figure 11, choose a large value for the pole term (B) to contribute significant phase lead [$P_P(\omega_{NC}')$] at the normalized desired gain crossover frequency of the system (ω_{NC}').
2. From Figure 12, read the value of the magnitude [$M_P(\omega_{NC}')$] for the chosen value of B at the normalized desired gain crossover frequency (ω_{NC}').
3. Determine the remaining phase lead [$P_Z(\omega_{NC}')$] which must be provided by the zero term (A) of the digital filter. This can be done by use of the equation:

$$P_Z(\omega_{NC}') = P_L - P_P(\omega_{NC}') \text{ (degrees)} \tag{53}$$

Where:

P_L = the required phase lead from the HCTL-1000's digital compensation filter (degrees)

$P_P(\omega_{NC}')$ = the phase lead contributed by the pole term (degrees)

4. On Figure 13, find the point where the normalized, desired gain crossover frequency (ω_{NC}') and the remaining phase lead [$P_Z(\omega_{NC}')$] intersect. Estimate the value of the term (A) at this point.
5. From Figure 14, find the value of the zero term's magnitude [$M_Z(\omega_{NC}')$] by plotting the estimated value of the zero (A) from step 4 at the normalized desired gain crossover frequency (ω_{NC}')

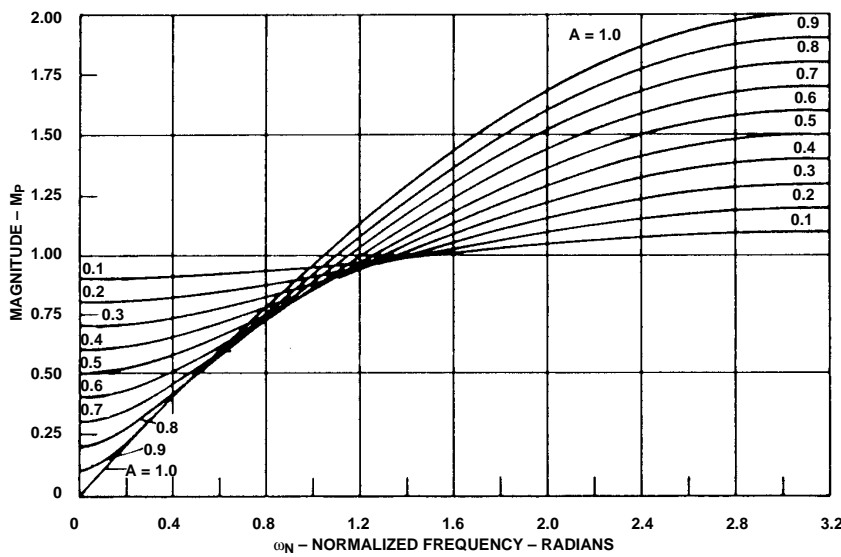


Figure 10. Magnitude Contribution of the Zero Term.

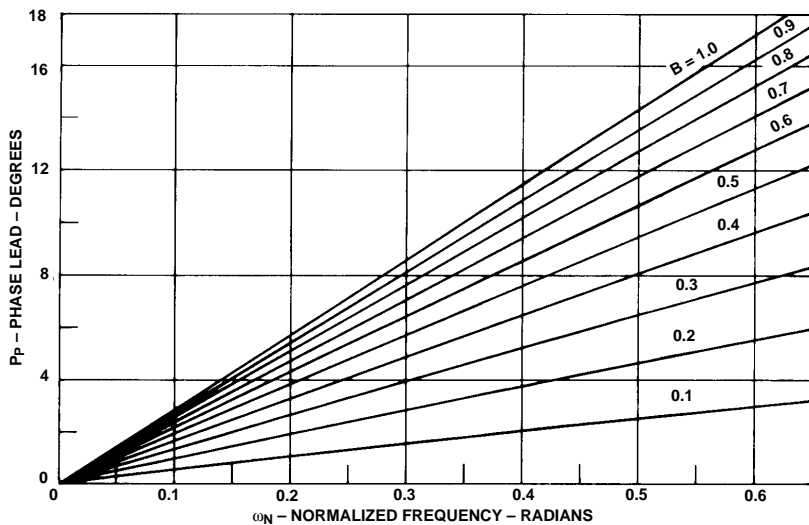


Figure 11. Phase Lead Contribution of the Pole Term.

and reading from the zero magnitude (M_Z) axis.

- Use the following equation to find the value of the gain (K):

$$K = \frac{K_F}{[M_Z(\omega_{NC}')][M_P(\omega_{NC}')]}$$

(54)

Where:

K_F = the gain required by the digital filter to provide the desired frequency (ω_{NC}')

$M_Z(\omega_{NC}')$ = the magnitude of the zero term

$M_P(\omega_{NC}')$ = the magnitude of the pole term

- Program the HCTL-1000 with:
 Register R20H = $256 \times A$ (55)
 Register R21H = $256 \times B$ (56)
 Register R22H = $4 \times K$ (57)

Note: The range of allowable values for registers 20H, 21 H, and 22H is integers between 0 and 255 decimal (0 to OFF Hex). If the required number for the gain that must be programmed into register R22H is greater than 255 decimal, then the gain of some other component in the system must be increased, or the desired gain crossover frequency (ω_C) must be decreased.

Appendices

Appendix A: Design Example of a System with a Voltage Source Amplifier:

In this section the HCTL-1000's digital filter parameters A, B, and K will be computed for an actual benchtop system with a voltage source amplifier.

Open Loop System Model:

The open loop system model for a system with a voltage source amplifier is shown in figure 6A.

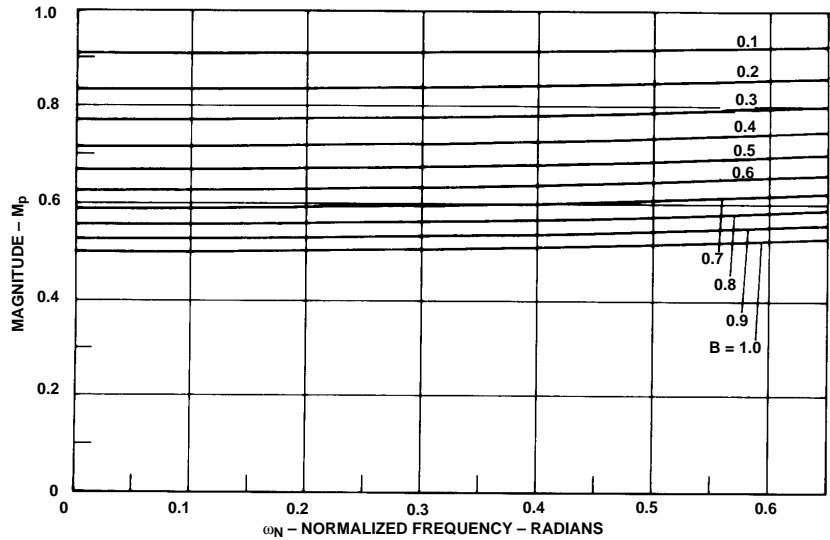


Figure 12. Magnitude Contribution of the Pole Term.

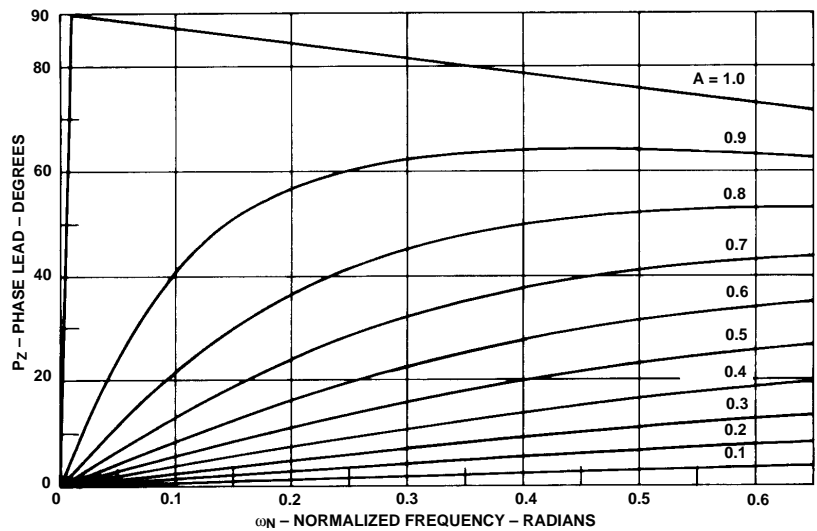


Figure 13. Phase Lead Contribution of the Zero Term.

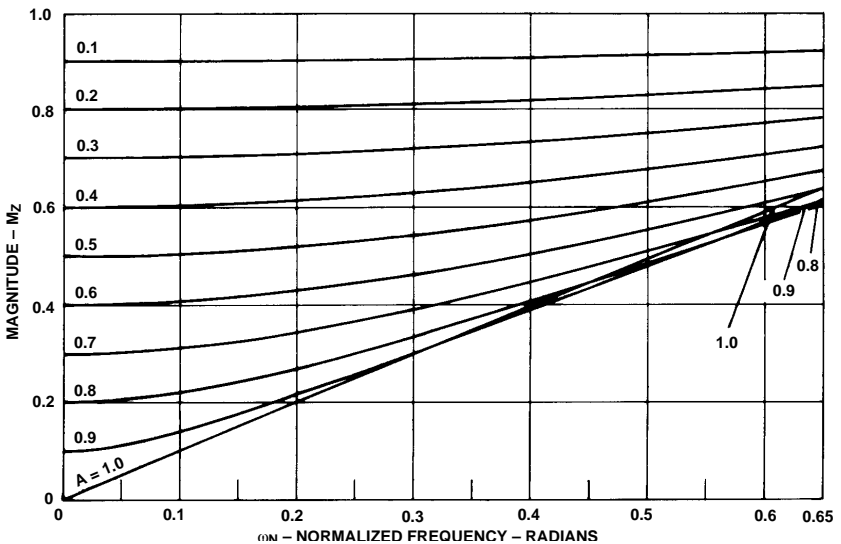


Figure 14. Magnitude Contribution of the Zero Term.

Choosing a bandwidth and sampling time:

The first step is to determine the desired bandwidth and sampling time of the system. For this example, a bandwidth of 60 Hz will be chosen. The sampling frequency of the HCTL-1000 should be at least 10 times the bandwidth of the system. The HCTL-1000's default sample time (t) of .00052 sec will be chosen. This makes the sampling frequency = $1/t = 1923$ Hz which is approximately 32 times the bandwidth of the system. See Equation 8.

The Zero Order Hold (ZOH) transfer function:

The transfer function of the zero order hold is:

$$Z(s) = e^{-st/2} = e^{-.00026s} \quad (6)$$

Where the time between samples (sampling time) (t) of the HCTL-1000 has been chosen as .52 milliseconds.

$$t = 0.00052 \text{ seconds}$$

The phase contribution of the zero order hold $[P_{ZOH}(\omega)]$ to the system is:

$$P_{ZOH}(\omega) = \frac{-[\omega][t]}{2} \quad (7)$$

Digital to Analog Converter (DAC) Parameters:

A linear, voltage source amplifier has been chosen to drive the motor. This amplifier requires a digital to analog converter (DAC) to convert the 8 bit motor command word from the HCTL-1000 into an analog voltage. The gain (K_D) of the example -5 to +5 volt DAC is as follows:

$$K_D = \frac{5 - (-5) \text{ volts}}{256 - 0 \text{ counts}} \quad (10)$$

$$K_D = .039 \text{ volts/count}$$

Amplifier Parameters:

Next, the gain of the voltage source amplifier must be determined. The gain of a linear, voltage source amplifier is determined by measuring the output voltage of the amplifier for a known input voltage. For the example system, the linear, voltage source amplifier's gain (K_A) is:

$$K_A = \frac{2.00 - 0 \text{ volts-out}}{1.00 - 0 \text{ volts-in}} \quad (11)$$

$$K_A = 2.00 \text{ volts-out/volt-in.}$$

Motor Parameters:

The motor parameters from the manufacturer's data sheet are:

Motor torque constant (K_T) =

$$3.15 \times 10^{-2} \text{ N-m/amp}$$

Motor moment of inertia (J_M) =

$$2.69 \times 10^{-6} \text{ kg-m}^2$$

Terminal resistance (R) = 5.44 ohms

Voltage constant (K_E) = 3.15×10^{-2} volt-sec/rad

Armature inductance (L) = 9.80×10^{-4} henries

System Moment of Inertia:

The total moment of inertia of the system (J) is determined by adding the moment of inertia of the motor's rotor (J_M), the moment of inertia of the load (J_L), and the moment of inertia of the encoder codewheel (J_C). For the example system there is no load external to the motor other than the encoder codewheel. Therefore, the total system moment of inertia (J) is:

$$J = J_M + J_L + J_C = (2.69 \times 10^{-6}) + 0 + (4 \times 10^{-8}) \text{ kg-m}^2 = 2.73 \times 10^{-6} \text{ kg-m}^2 \quad (12)$$

The DC Motor Transfer Function:

The motor transfer function of a DC motor driven by a voltage source amplifier is:

$$G(s) = \frac{\theta(s)}{v(s)} = \frac{\text{position output}}{\text{voltage input}} = \frac{K_{MV}}{s[sT_M + 1][sT_E + 1]} \quad (13)$$

Calculation of the Transfer Function of the DC Motor:

The motor parameters and the total moment of inertia of the system are used to calculate the mechanical and electrical time constants and the gain constant of the motor. The equations for the mechanical time constant (T_M) and the electrical time constant (T_E) of the motor are:

$$T_M = \frac{[R][J]}{[K_E][K_T]} \text{ (seconds)} \quad (14)$$

$$T_E = \frac{L}{R} \text{ (seconds)} \quad (15)$$

The gain constant (K_{MV}) of the motor is computed as:

$$K_{MV} = \frac{1}{K_E} \text{ (rad/amp/-sec}^2\text{)} \quad (16)$$

For the example system:

$$T_M = \frac{[5.44][2.73 \times 10^{-6}]}{[.0315][.0315]} \text{ (seconds)}$$

$$T_M = .015 \text{ seconds}$$

$$T_E = \frac{9.8 \times 10^{-4}}{5.44} \text{ (seconds)}$$

$$T_E = 0.18 \text{ milliseconds}$$

$$K_{MV} = \frac{1}{.0315} \text{ rad/sec/volt}$$

$$K_{MV} = 31.75 \text{ rad/sec/volt}$$

Because the mechanical time constant (T_M) is more than 10 times larger than the electrical time constant (T_E) of the example, the DC motor transfer reduces to:

$$G(s) = \frac{K_{MV}}{s(sT_M + 1)} = \frac{31.75}{s(0.15s + 1)} \quad (17)$$

The phase contribution $[P_M(\omega)]$ of the motor driven at frequency (ω) by a voltage source amplifier to the system is:

$$P_M(\omega) = -\arctan(\omega T_M) - \frac{\pi}{2} \quad (\text{radians})$$

$$P_M(\omega) = -\arctan(\omega[.015]) - \frac{\pi}{2} \quad (\text{radians}) \quad (18)$$

Magnitude contribution $[M_M(\omega)]$ of the motor driven at frequency (ω) by a voltage source amplifier is:

$$M_M(\omega) = \frac{K_{MV}}{\omega \sqrt{1 + ([\omega] [T_M])^2}} \quad (19)$$

$$M_M(\omega) = \frac{31.75}{\omega \sqrt{1 + ([\omega] [.015])^2}}$$

Encoder Parameters:

The parameters of the encoder must be determined. The parameters of the Agilent Technologies HEDS-5000 encoder chosen for this example are:

$$N = 256 \text{ slits in the codewheel}$$

$$C = 4N = 1024 \text{ quadrature encoder counts/ codewheel revolution}$$

$$J_C = 4 \times 10^{-8} \text{ kg}\cdot\text{m}^2$$

Therefore, the transfer function and magnitude contribution (E) of the encoder to the system is:

$$E = \frac{C}{2\pi} = \frac{1024}{2\pi} \quad (\text{count/radians}) \quad (24)$$

Calculation of the Open Loop Transfer Function:

The open loop transfer function is calculated as:

$$M(s) = [K_D] [ZOH] [K_A] [G(s)] [C/(2 \times \pi)] \quad (5)$$

$$M(s) = [.039][e^{-.00026s}][2.0] \left[\frac{31.75}{s(.015s + 1)} \right] \left[\frac{1024}{2\pi} \right]$$

$$M(s) = \frac{403.56 e^{-.00026s}}{s(.015s + 1)}$$

Determination of the Gain Crossover Frequency, Phase Crossover Frequency, Gain Margin, and Phase Margin of the Open Loop Transfer Function Using a Bode Plot:

The equation which is used to find the phase $[P_{HU}(\omega)]$ of the uncompensated, open loop transfer function is:

$$P_{HU}(\omega) = P_M(\omega) + P_{ZOH}(\omega) \quad (\text{radians}) \quad (27)$$

$$P_{HU}(\omega) = -\arctan(\omega \times T_M) - \frac{\pi}{2} - \frac{\omega t}{2} \quad (\text{radians}) \quad (29)$$

$$P_{HU}(\omega) = -\arctan(\omega \times .015) - \frac{\pi}{2} - \frac{\omega[.00052]}{2} \quad (\text{rad})$$

The phase $[P_{HU}(\omega)]$ can now be calculated for various values of frequency (ω) . To be graphed onto a Bode plot, the values for the phase $[P_{HU}(\omega)]$ must be converted from radians to degrees using the equation:

$$1 \text{ radian} = 57.296 \text{ degrees} \quad (31)$$

Some values for the phase $[P_{HU}(\omega)]$ are shown in degrees for various values of frequency (ω) in Table 2.

The equation which is used to find the magnitude $[M_U(\omega)]$ of the uncompensated, open loop transfer function is:

$$M_U(\omega) = [M_U(\omega)][K_D][K_A][E] \quad (32)$$

$$M_U(\omega) = \frac{[K_{MV}][K_D][K_A][C/(2\pi)]}{\omega \sqrt{1 + ([T_M][\omega])^2}}$$

$$M_U(\omega) = \frac{[[31.75][.039][2.0][(1024/(2\pi))]]}{\omega \sqrt{1 + ([.015][\omega])^2}}$$

The magnitude $[M_U(\omega)]$ can now be calculated for various values of frequency (ω) . To be graphed onto a Bode plot, the values for the magnitude $[M_U(\omega)]$ must be converted from decimal to decibels using the equation:

$$\text{db} = 20 \log [M_U(\omega)] \quad (36)$$

Some values for the magnitude $[M_U(\omega)]$ are shown in decibel for various values of frequency (ω) in Table 2.

Figure 15 shows a Bode plot of the phase $[P_{HU}(\omega)]$ and gain $[M_U(\omega)]$.

The next step is to find the gain and phase crossover frequencies of the uncompensated, open loop system. The gain crossover frequency (ω_C) of the uncompensated open loop system can be computed by setting the magnitude $[M_U(\omega)]$ in db equal to 0 db and solving the equation for (ω_C) .

$$20 \log [M_U(\omega_C)] = 0 \text{ db} \quad (38)$$

$$20 \log \left[\frac{[K_{MV}][K_D][K_A][C/(2\pi)]}{\omega_C \sqrt{1 + ([\omega_C][T_M])^2}} \right] = 0 \text{ db}$$

The difficulty in choosing this method is that the solution can involve a fourth order (quartic) equation. A simpler method is to read the gain crossover frequency from the Bode plot

Table 2. The magnitude (M_U) and phase (P_{HU}) of the open loop transfer function over frequency (ω), for the system with a voltage source amplifier.

Frequency (ω) (radians/sec)	Magnitude (M_U) (decibels)	Phase (P_{HU}) (degrees)
1	52.12	-90.87
2	46.09	-91.75
3	42.57	-92.62
4	40.06	-93.49
5	38.11	-94.36
6	36.52	-95.22
7	35.17	-96.09
8	33.99	-96.95
9	32.95	-97.81
10	32.02	-98.66
20	25.72	-106.97
30	21.78	-114.63
40	18.74	-121.51
50	16.21	-127.56
60	13.98	-132.82
70	12.00	-137.38
80	10.19	-141.33
90	8.54	-144.76
100	7.01	-147.75
200	-3.89	-164.51
300	-10.68	-171.92
400	-15.59	-176.48
500	-19.42	-179.84
600	-22.57	-182.59
700	-25.23	-184.98
800	-27.54	-187.15
900	-29.58	-189.16
1000	-31.41	-191.08

shown in figure 15. From the Bode plot it can be seen that the magnitude $[M_U(\omega)]$ crosses the 0 db axis at approximately 160 radians/sec. Therefore the gain crossover frequency (ω_C) of the uncompensated, open loop system is:

$$\omega_C = 160 \text{ radians/sec} \\ = 25.5 \text{ Hz}$$

The phase crossover frequency (ω_p) of the uncompensated, open loop system can be found by setting the phase $[P_{HU}(\omega)]$ equal to $-\pi$ radians (i.e. -180 degrees) and solving for (ω_p):

$$P_{HU}(\omega_p) = -\pi \text{ radians} - \arctan \left(\frac{[\omega_p][T_M]}{2} \right) \\ - \frac{\pi}{2} - \frac{[\omega_p][t]}{2} = -\pi \text{ radians} \quad (37)$$

As with the phase crossover frequency calculation, the simplest method to find the phase crossover frequency (ω_p) is to read it directly from the Bode plot. The phase crossover frequency (ω_p) is the point where the phase $[P_{HU}(\omega)]$ crosses the -180 degree axis. From the Bode plot shown in figure 15 it can be seen that the phase crossover frequency (ω_p) is approximately:

$$\omega_p = 500 \text{ radians/sec} = 79.6 \text{ Hz}$$

The gain margin (G_M) is computed at the phase crossover frequency (ω_p) as:

$$G_M = -20 \log [M_U(\omega_p)] \text{ db} \quad (40)$$

$$G_M = -20 \log \left[\frac{[K_{MV}][K_D][K_A][C/(2\pi)]}{\omega_p \sqrt{1 + ([\omega_p][T_M])^2}} \right] \text{ db}$$

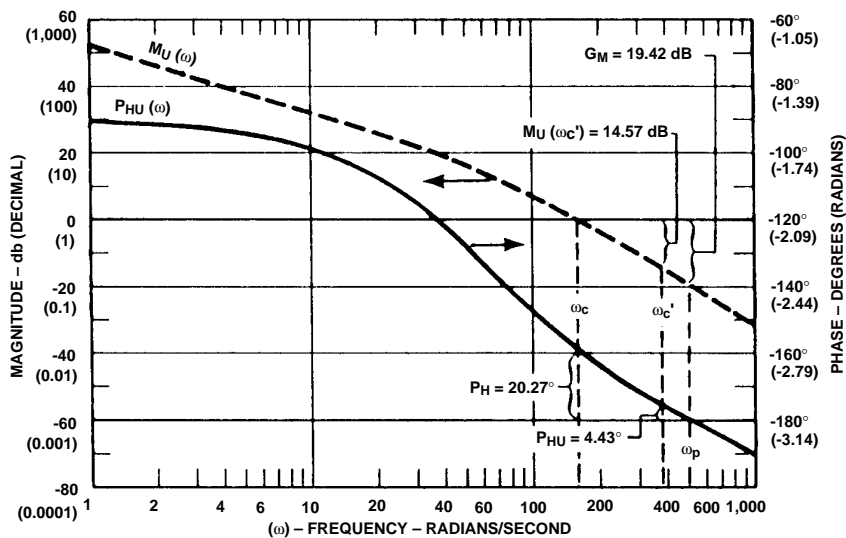


Figure 15. Bode Plot of the Open Loop System with the Voltage Source Before Compensation.

$$= -20 \log \left[\frac{[[31.75][.039][2.0][(1024/(2\pi))]]}{500 \times \sqrt{1 + ([500][.015])^2}} \right] \text{ db}$$

$$G_M = 19.42 \text{ db}$$

The gain margin (G_M) can also be read directly from the Bode plot as the difference between the 0 db axis and the magnitude $[M_U(\omega_P)]$ at the phase crossover frequency (ω_P).

The phase margin (P_H) is computed as 180 degrees plus the phase $[P_{HU}(\omega_C)]$ in degrees at the gain crossover frequency (ω_C).

$$P_H = [180 \text{ deg.} + [57.296(\text{deg/rad}) [P_{HU}(\omega_C)]] \text{ degrees} \quad (41)$$

$$= 180 + 57.296 \left(-\arctan([\omega_C]$$

$$[T_M]) - \frac{\pi}{2} - \frac{[\omega_C] [t]}{2} \right) \text{ deg}$$

$$= 180 + 57.296 \left(-\arctan([160]$$

$$[.015]) - \frac{\pi}{2} - \frac{[160][.00052]}{2} \right)$$

$$= 20.27 \text{ degrees}$$

The phase margin (P_H) can also be read directly from the Bode plot as the difference between the phase $[P_{HU}(\omega)]$ at the gain crossover frequency (ω_C) and the -180 degree axis.

Modification of the Open Loop System Transfer Function Using Compensation to Achieve the Desired Closed Loop Characteristics:

Now that the uncompensated, open loop characteristics of the system are known and plotted on a Bode plot, the HCTL-1000's digital compensation filter can be

designed to provide the desired closed loop characteristics.

For this system closed loop bandwidth (BW) of 60 Hertz [377 radians/sec] is desired, and a compensated, desired phase margin (P_{HC}) of 40 degrees has been chosen.

The desired open loop gain crossover frequency (ω_C') can be approximated as the desired closed loop bandwidth (BW):

$$(\omega_C') = \text{BW radians/sec} \quad (42)$$

$$(\omega_C') = 377 \text{ radians/sec}$$

The normalized, desired gain crossover frequency (ω_{NC}') is:

$$\begin{aligned} \omega_{NC}' &= [\omega_C'] [t] \text{ radians} \quad (48) \\ \omega_{NC}' &= [377 \text{ rad/sec}] [.00052 \text{ sec}] \\ &= .1960 \text{ radians} \end{aligned}$$

Determination of the phase margin and gain of the open loop transfer function at the desired, open loop gain crossover frequency:

For the example system with the voltage source amplifier, where the electrical time constant has been neglected, the phase margin (P_{MU}) of the uncompensated system at the desired gain crossover frequency (ω_C') is computed as:

$$P_{MU} = [180 \text{ deg.} + ([57.296 (\text{deg/rad})][P_{HU}(\omega_C')]] \text{ degrees} \quad (43)$$

$$P_{MU} = 180 + [57.296][-\arctan([\omega_C'] [T_M]) - \left[\frac{\pi}{2} - \frac{[\omega_C] [t]}{2} \right]]$$

$$P_{MU} = 180 + [57.296][-\arctan$$

$$\left([377][.015] \right) - \left[\frac{3.142}{2} - \frac{[377][.00052]}{2} \right]$$

$$P_{MU} = 4.43 \text{ degrees}$$

The phase lead (P_L) that the HCTL-1000's digital filter must provide to achieve the desired closed loop performance is the difference between the desired, compensated phase margin (P_{HC}) at the gain crossover frequency (ω_C) and the uncompensated phase margin (P_{MU}) at frequency (ω_C'):

$$P_L = P_{HC} - P_{MU} \text{ degrees} \quad (44)$$

$$P_L = 40 - 4.43 = 35.57 \text{ degrees}$$

The magnitude $[M_U(\omega_C')]$ of the uncompensated system at frequency (ω_C') with a voltage source amplifier where the electrical time constant has been neglected is:

$$M_U(\omega_C') =$$

$$\frac{[K_{MV}][K_D][K_A][C/(2 \times \pi)]}{\omega_C' \sqrt{1 + ([T_M][\omega_C'])^2}} \quad (34)$$

$$M_U(\omega_C') =$$

$$\frac{[[31.74][.039][2.0][1024/([2][3.142)]]}{377 \sqrt{1 + ([.015][377])^2}}$$

$$M_U(\omega_C') = .186$$

$$(-20 \log .186 = 14.6 \text{ db})$$

The gain (K_F) required from the digital filter to make the overall gain of the system equal to 1 at the desired gain crossover frequency (ω_C') is defined as:

$$K_F = \frac{1}{M_U(\omega_C')} \quad (45)$$

$$K_F = \frac{1}{.186}$$

$$K_F = 5.38$$

Use of the Combination Method to Design the HCTL-1000's Digital Compensation Filter:

On Figure 16 the plot of $B = 0.9$ is chosen to contribute 5.3 degrees of phase lead ($P_P(\omega_{NC}')$) at $\omega_{NC}' = 0.196$.

The plot of $B = 0.9$ in figure 17 at $\omega_{NC}' = 0.196$ gives a gain [$M_P(\omega_{NC}')$] of 0.53.

The remaining phase lead at frequency (ω_C') which must be contributed by the zero term [$P_Z(\omega_{NC}')$] is:

$$P_Z(\omega_{NC}') = P_L - P_P(\omega_{NC}') \quad (53)$$

$$= 35.57 - 5.3 = 30.27 \text{ degrees}$$

On figure 18, the point where $\omega_{NC}' = .196$ and $P_Z(\omega_{NC}') = 30.27$ intersect is $A = 0.76$.

The magnitude of the zero term where $A = .76$ at $\omega_{NC}' = 0.196$ on figure 19 is $M_Z(\omega_{NC}') = 0.29$. The gain (K) which must be contributed by the digital filter is:

$$K = \frac{K_F}{[M_P(\omega_{NC}')][M_Z(\omega_{NC}')]}$$

$$= \frac{5.38}{[0.53][.295]} = 34.4 \quad (54)$$

The values which should be programmed into the HCTL-1000 are therefore:

$$A = [256][0.76] = 194.56 \cong 195 \quad (55)$$

$$B = [256][0.9] = 230.4 \cong 230 \quad (56)$$

$$K = [4][34.4] = 137.6 \cong 138 \quad (57)$$

Figure 20 shows the actual response of the closed loop system to a step input of 25 quadrature counts. (25 quadrature counts on a 256 count codewheel that is

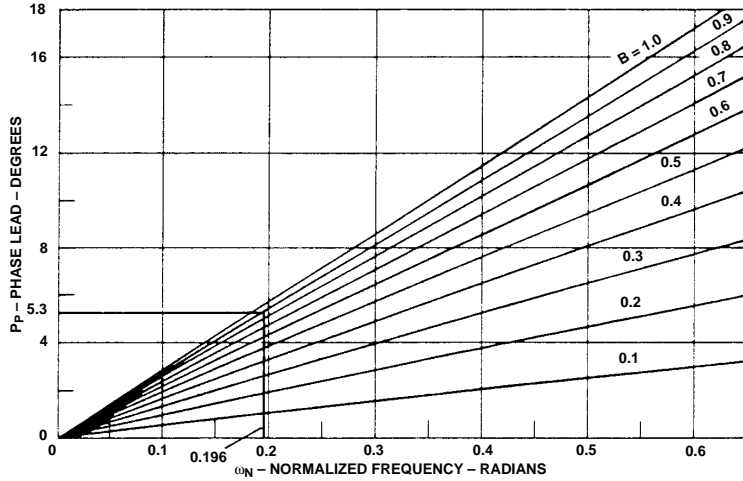


Figure 16. Phase Lead Contribution of the Pole Term to the Closed Loop System Compensator.

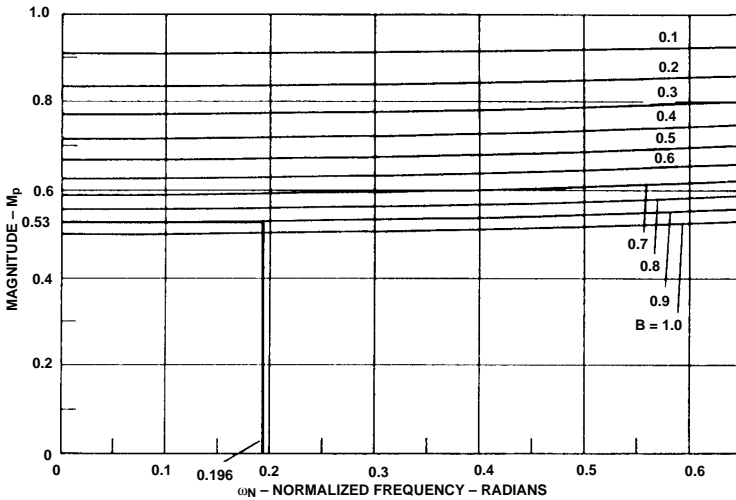


Figure 17. Magnitude Contribution of the Pole Term to the Closed Loop System Compensator.

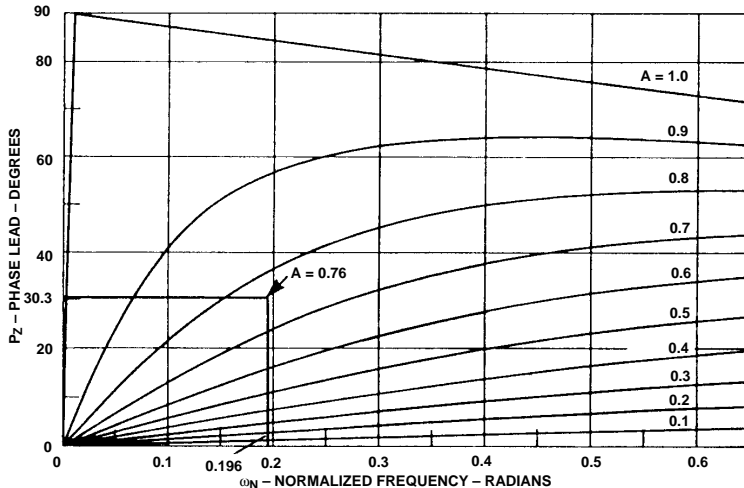


Figure 18. Phase Lead Contribution of the Zero Term to the Closed Loop System Compensator.

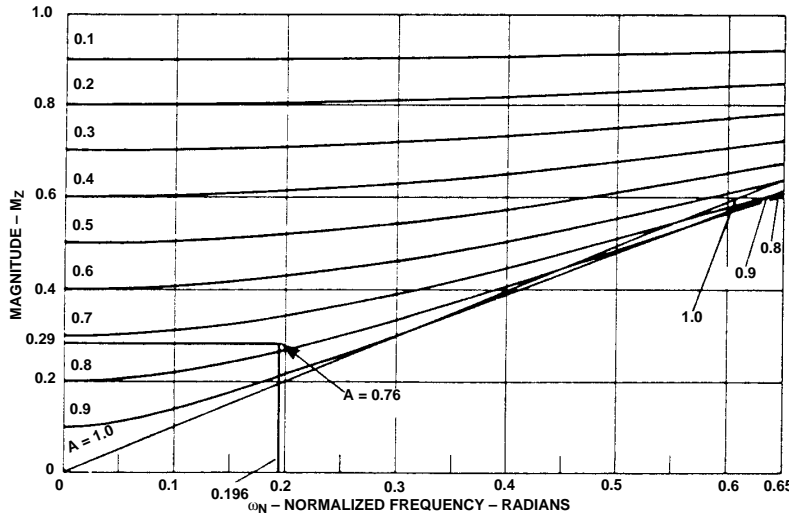


Figure 19. Magnitude Contribution of the Zero Term to the Closed Loop System Compensator.

multiplied by 4 equals 8.79 degrees of angular rotation). The amount of overshoot of the system's response is roughly consistent with a system with 40 degrees of phase margin.

For high phase margin systems the relationship between the risetime (t_C) and the bandwidth (BW) of a closed loop system can be approximated as:

$$t_R = \frac{2.2}{\text{BW (in radians/sec)}} \quad (58)$$

For low phase margin systems which have some overshoot in the closed loop step response, the relationship between the risetime (t_R) and the bandwidth (BW) of a closed loop system can be approximated as:

$$\frac{0.6}{\text{BW}} < t_R < \frac{2.2}{\text{BW}} \quad (59)$$

Figure 21 shows the actual rise time of the closed loop system to a step input of 25 quadrature counts. Note that the risetime is defined as the time required for the output to go from 10% to 90% of its final value. Since this sys-

tem has some overshoot, equation (59) applies. The risetime (t_R) should therefore be:

$$\frac{0.6}{[60][6.28]} < t_R < \frac{2.2}{[60][6.28]} \text{ seconds}$$

$$.00159 < t_R < .0058 \text{ seconds}$$

The risetime measured from figure 21 is:

$$t_R = 0.0042 \text{ seconds}$$

Therefore the measured risetime falls within the expected range of values for a bandwidth of 60 Hertz.

Appendix B: Design Example of a System With a Current Source Amplifier:

In this section the HCTL-1000's digital filter parameters A, B, and K will be computed for an actual benchtop system with a current source amplifier.

Open Loop System Model:
The open loop system model for a system with a current source amplifier is shown in figure 6B.

Choosing a bandwidth and sampling time:

The first step is to determine the desired bandwidth and sampling time of the system. For this example, a bandwidth of 60 Hz will be chosen. The sampling frequency of the HCTL-1000 should be at least 10 times the bandwidth of the system. The HCTL-1000's default sample time of .00052 sec will be chosen. This makes the sampling frequency = $1/t = 1923$ Hz which is approximately 32 times the bandwidth of the system. See Equation 8.

The zero order hold (ZOH) transfer function:

The transfer function of the zero order hold is:

$$Z(s) = e^{-st/2} = e^{-.00026s} \quad (6)$$

Where the time between samples (sampling time) (t) of the HCTL-1000 has been chosen as .52 milliseconds.

$$t = 0.00052 \text{ seconds}$$

The phase contribution of the zero order hold ($P_{ZOH}(\omega)$) to the system is:

$$P_{ZOH}(\omega) = - \frac{[\omega][t]}{2} \quad (7)$$

Amplifier parameters:

Next, the gain of the current source amplifier must be determined. For the example system, a pulse width modulated, current source amplifier with a gain (K_A) = 0.02 amps-out/count has been chosen. No digital to analog converter (DAC) is used with a pulse width modulated amplifier.

$$K_A = 0.02 \text{ amps-out/count} \quad (11)$$

Motor parameters:

The motor parameters from the manufacturer's data sheet are:

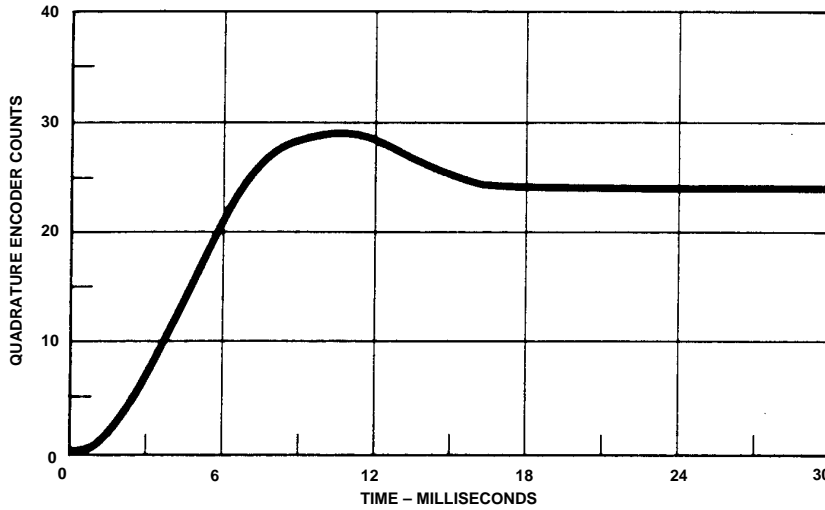


Figure 20. Response of the Closed Loop System to a Step Input of 25 Quadrature Encoder Counts.

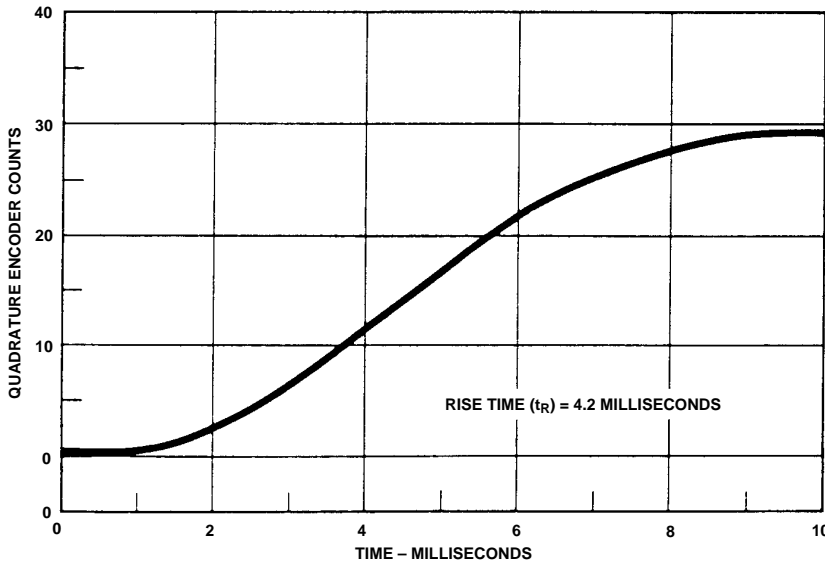


Figure 21. Rise Time of the Closed Loop System to a Step Input of 25 Quadrature Encoder Counts.

Motor torque constant (K_T) =
 3.15×10^{-2} N-m/amp

Motor moment of inertia (J_M) =
 2.69×10^{-6} kg-m²

Terminal resistance (R) = 5.44
 ohms

Voltage constant (K_E) = $3.15 \times$
 10^{-2} volt-sec/rad

Armature inductance (L) = 9.8
 $\times 10^{-4}$ henries

System moment of inertia:

The total moment of inertia of the system (J) is determined by adding the moment of inertia of the motor's rotor (J_M), the moment of inertia of the load (J_L), and the moment of inertia of the encoder codewheel (J_C). For the example system there is no load external to the motor other than the encoder codewheel. Therefore, the total system moment of inertia (J) is:

$$J = J_M + J_L + J_C = (2.69 \times 10^{-6}) + 0 + (4 \times 10^{-8}) \text{ kg-m}^2 \quad (12)$$

$$= 2.73 \times 10^{-6} \text{ kg-m}^2$$

The DC motor transfer function:

The motor transfer function of a DC motor driven by a current source amplifier is:

$$G(s) = \frac{\theta(s)}{v(s)} = \frac{\text{position output}}{\text{voltage input}}$$

$$= \frac{K_{MC}}{s^2} \quad (20)$$

Calculation of the gain constant and transfer function of the DC motor:

The motor parameters and the total moment of inertia of the system including the motor are used to calculate the gain constant of the motor. The gain constant (K_{MC}) of the motor is computed as:

$$K_{MC} = \frac{K_T}{J} \text{ (rad/amp-sec}^2\text{)} \quad (21)$$

For the example system:

$$K_{MC} = \frac{.0315 \text{ N-m/amp}}{2.73 \times 10^{-6} \text{ kg-m}^2}$$

$$= 11,538.46 \text{ rad/amp-sec}^2$$

Therefore the motor transfer function is:

$$G(s) = \frac{K_{MC}}{s^2} = \frac{11,538.46}{s^2}$$

The phase contribution ($P_M(\omega)$) of the motor driven at frequency (ω) by a current source amplifier to the system is:

$$P_M(\omega) = -\pi \text{ (radians)} \quad (22)$$

The magnitude contribution ($M_M(\omega)$) of the motor driven at frequency (ω) by a current source amplifier is:

$$M_U(\omega) = \frac{K_{MC}}{(\omega)^2} \quad (23)$$

$$M_U(\omega) = \frac{11,538.46}{(\omega)^2}$$

Encoder parameters:

The parameters of the encoder must be determined. The parameters of the Agilent Technologies HEDS-5000 encoder chosen for this example are:

$N = 256$ slits in the codewheel

$C = 4N = 1024$ quadrature encoder counts/codewheel revolution

$J_C = 4 \times 10^{-8}$ kg-m²

Therefore, the transfer function and magnitude contribution (E) of the encoder to the system is:

$$E = \frac{C}{2\pi} = \frac{1024}{2\pi} \text{ (counts/radian)} \quad (24)$$

Calculation of the open loop transfer function:

The open loop transfer function is calculated as:

$$M(s) = [ZOH][K_A][G(s)][C/(2\pi)]$$

$$M(s) = [e^{-0.00026s}][0.02] \quad (5)$$

$$\left[\frac{11,538.46}{s^2} \right] \left[\frac{1024}{2\pi} \right]$$

$$M(s) = \frac{37,610[e^{-0.00026s}]}{s^2}$$

Determination of the gain crossover frequency, phase crossover frequency, gain margin, and phase margin of the open loop transfer function using a Bode plot:

The equation which is to be used to find the phase $[P_{HU}(\omega)]$ of the uncompensated, open loop transfer function is:

$$P_{HU}(\omega) = P_M(\omega) + P_{ZOH}(\omega) \text{ (radians)} \quad (27)$$

$$P_{HU}(\omega) = -\pi - \frac{[\omega][t]}{2} \text{ (radians)} \quad (30)$$

$$P_{HU}(\omega) = -\pi - \frac{[\omega][.00052]}{2} \text{ (radians)}$$

The phase $[P_{HU}(\omega)]$ can now be calculated for various values of frequency (ω). To be graphed into a Bode plot, the values for the phase $[P_{HU}(\omega)]$ must be converted from radians to degrees using the equation:

$$1 \text{ radian} = 57.296 \text{ degrees} \quad (31)$$

Some values for the phase $[P_{HU}(\omega)]$ are shown in degrees for various values of frequency (ω) in Table 3. The equation which is used to find the magnitude $[M_U(\omega)]$ of the uncompensated, open loop transfer function is:

$$M_U(\omega) = [M_M(\omega)][K_A][E] \quad (32)$$

$$M_U(\omega) = \frac{[K_{MC}][K_A][C/2\pi]}{\omega^2}$$

$$M_U(\omega) = \frac{11,538.46 \times 0.02 \times (1024/(2\pi))}{\omega^2}$$

The magnitude $[M_U(\omega)]$ can now be calculated for various values of frequency (ω). To be graphed onto a Bode plot, the values for the magnitude $[M_U(\omega)]$ must be converted from decimal to decibels using the equation:

$$\text{db} = 20 \log [M_U(\omega)] \quad (36)$$

Some values for the magnitude $[M_U(\omega)]$ are shown in decibel for various values of frequency (ω) in Table 3.

Figure 22 shows a Bode plot of the phase $[P_{HU}(\omega)]$ and gain $[M_U(\omega)]$.

The next step is to find the gain and phase crossover frequencies of the uncompensated, open loop system. The gain crossover fre-

quency (ω_C) of the uncompensated open loop system can be computed by setting the magnitude $[M_U(\omega)]$ in db equal to 0 db and solving the equation for (ω_C):

$$20 \log [M_U(\omega_C)] = 0 \text{ db} \quad (38)$$

$$20 \log = \left[\frac{[K_{MC}][K_A][C/2\pi]}{(\omega_C)^2} \right] = 0 \text{ db}$$

$$\omega_C = \sqrt{[K_{MC}][K_A][C/(2\pi)]}$$

Another method is to read the gain crossover frequency from the Bode plot shown in Figure 22. From the Bode plot it can be seen that the magnitude $[M_U(\omega)]$ crosses the 0 db axis at approximately 195 radians/sec. Therefore gain crossover frequency (ω_C) of the uncompensated, open loop system is:

$$\omega_C = 195 \text{ radians/sec} = 31.04 \text{ Hz}$$

The phase crossover frequency (ω_P) of the uncompensated, open loop system can be found by setting the phase $[P_{HU}(\omega)]$ equal to $-\pi$ radians (-180 degrees) and solving for (ω_P):

$$P_{HU}(\omega_P) = -\pi \text{ radians} \quad (37)$$

$$-\pi - \frac{[\omega][t]}{2} = -\pi \text{ radians}$$

Another method to find the phase crossover frequency (ω_P) is to read it directly from the Bode plot. The phase crossover frequency (ω_P) is the point where the phase crosses the -180 degree axis. From the Bode plot shown in figure 22 it can be seen that the phase crossover frequency (ω_P) is:

$$(\omega_P) = 0.0 \text{ radians/sec} = 0.0 \text{ Hz}$$

Table 3. The magnitude (M_U) and phase (P_{HU}) of the open loop transfer function over frequency (ω), for the system with a voltage source amplifier.

Frequency (ω) (radians/sec)	Magnitude (M_U) (decibels)	Phase (P_{HU}) (degrees)
1	91.61	-180.02
2	79.57	-180.03
3	72.52	-180.05
4	67.53	-180.06
5	63.65	-180.08
6	60.48	-180.09
7	57.81	-180.10
8	55.49	-180.12
9	53.44	-180.13
10	51.61	-180.15
20	39.57	-180.30
30	32.52	-180.45
40	27.53	-180.60
50	23.65	-180.75
60	20.48	-180.89
70	17.81	-181.04
80	15.49	-181.19
90	13.44	-181.34
100	11.61	-181.49
200	-4.3	-182.98
300	-7.48	-184.47
400	-12.47	-185.96
500	-16.35	-187.45
600	-19.52	-188.94
700	-22.19	-190.43
800	-24.51	-191.92
900	-26.56	-193.41
1000	-28.39	-194.90

The gain margin (G_M) is computed at the phase crossover frequency (ω_P) as:

$$G_M = -20 \log [M_U(\omega_P)] \text{ db} \quad (40) = [180 + 57.296]$$

However, since the phase crossover frequency is zero, the system is unstable, and the gain margin is minus infinity.

The phase margin (P_H) can be computed as 180 degrees plus the phase [$P_{HU}(\omega)$] in degrees at the gain crossover frequency (ω_C).

$$P_H = [180 \text{ deg.} + [57.296 \text{ (deg/rad)}][P_{HU}(\omega_C)]] \text{ degrees} \quad (41)$$

$$= 180 + [57.296]$$

$$\left[-\pi - \frac{[\omega_C] [t]}{2} \right] \text{ degrees}$$

$$\left[-\pi - \frac{[195][.00052]}{2} \right] = -2.9 \text{ degrees (ie. unstable!)} \quad (42)$$

The phase margin (P_H) can also be read directly from the Bode plot as the difference between the phase [$P_{HU}(\omega)$] at the gain crossover frequency (ω_C) and the -180 degree axis.

Modification of the Open Loop System Transfer Function Using Compensation to Achieve the Desired Closed Loop Characteristics.

Now that the uncompensated, open loop characteristics of the system are known and plotted on a Bode plot, the HCTL-1000's digital compensation filter can be designed to provide the desired closed loop characteristics.

For this system closed loop bandwidth (BW) of 60 Hertz [377 radians/sec] is desired, and a compensated, phase margin (P_{HC}) of 40 degrees has been chosen.

The desired open loop gain crossover frequency (ω_C') can be approximated as the desired closed loop bandwidth (BW):

$$\omega_C' = \text{BW radians/sec} \quad (42)$$

$$\omega_C' = 377 \text{ radians/sec}$$

The normalized, desired gain crossover frequency (ω_{NC}') is:

$$\omega_{NC}' = [\omega_C'] [t] \text{ radians} \quad (48)$$

$$\omega_{NC}' = [377 \text{ (rad/sec)}] [.00052 \text{ (sec)}] = .1960 \text{ radians}$$

Determination of the phase margin and gain of the open loop transfer function at the desired gain crossover frequency:

For the example system with the current source amplifier the phase margin (P_{MU}) of the uncompensated system at the desired gain crossover frequency (ω_C') is computed as:

$$P_{MU} = [180 \text{ deg.} + [57.296 \text{ (deg/rad)}][P_{HU}(\omega_C')]] \text{ degrees} \quad (43)$$

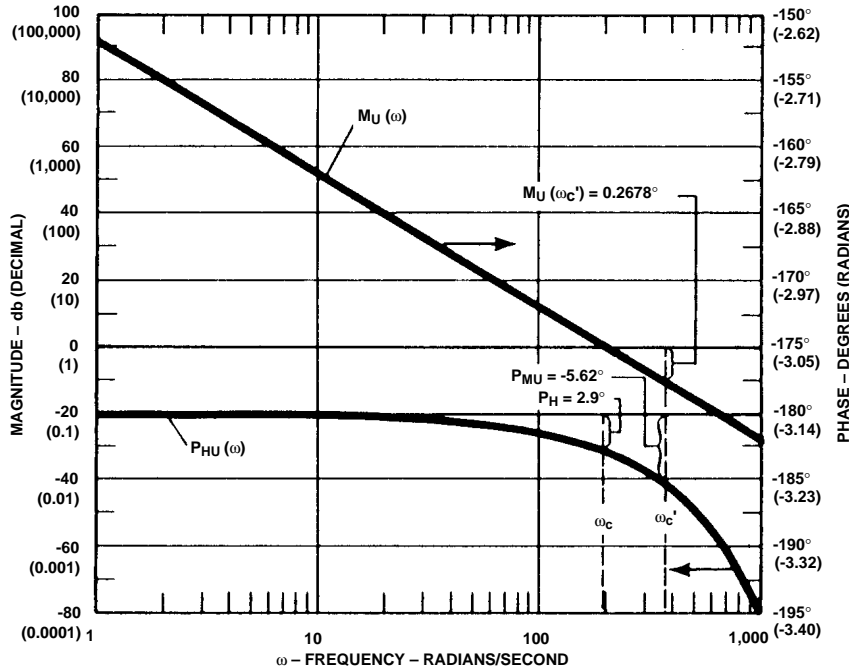


Figure 22. Bode Plot of the Open Loop System with the Current Source Before Compensation.

$$P_{MU} = 180 + 57.296$$

$$\left[-\pi - \frac{[\omega_c'] [t]}{2} \right]$$

$$P_{MU} = 180 + 57.296$$

$$\left[-\pi - \frac{[377][.00052]}{2} \right]$$

$$P_{MU} = -5.62 \text{ degrees}$$

The phase lead (P_L) that the HCTL-1000's digital filter must provide to achieve the desired closed loop performance is the difference between the desired, compensated phase margin (P_{HC}) at the gain crossover frequency (ω_c) and the uncompensated phase margin (P_{MU}) at frequency (ω_c):

$$P_L = P_{HC} - P_{MU} \text{ degrees} \quad (44)$$

$$P_L = 40 - (-5.62) = 45.62 \text{ degrees}$$

The magnitude [$M_U(\omega_c')$] of the uncompensated system at

frequency (ω_c') with a current source amplifier is:

$$M_U(\omega) = \frac{[K_M][K_A][C/(2\pi)]}{(\omega_c')^2} \quad (35)$$

$$M_U(\omega) = \frac{[11,538.46] \times [0.02] \times [1024/(2 \times 3.1416)]}{(377)^2}$$

$$M_U(\omega) = .265$$

The gain (K_F) required from the digital filter to make the overall gain of the system equal to 1 at the desired gain crossover frequency (ω_c') is defined as:

$$K_F = \frac{1}{M_U(\omega_c')} \quad (45)$$

$$K_F = \frac{1}{.265}$$

$$K_F = 3.77$$

Use of the Combination Method to Design the HCTL-1000's Digital Compensation Filter:

On figure 23 the plot of $B = 0.9$ is chosen to contribute 5.3 degrees of phase lead [$P_P(\omega_{NC}')$] at $\omega_{NC}' = 0.196$.

The plot of $B = 0.9$ in figure 24 at $\omega_{NC}' = 0.196$ gives a gain [$M_P(\omega_{NC}')$] of 0.53.

The remaining phase lead at frequency (ω_c') which must be contributed by the zero term [$P_Z(\omega_{NC}')$] is:

$$P_Z(\omega_{NC}') = P_L - P_P(\omega_{NC}') \quad (53)$$

$$= 45.62 - 5.3 = 40.32 \text{ degrees}$$

On figure 25, the point where $\omega_{NC}' = .196$ and $P_Z(\omega_{NC}') = 40.32$ intersect is $A = 0.826$.

The magnitude of the zero term where $A = .826$ at $\omega_{NC}' = 0.196$ on figure 26 is $M_Z(\omega_{NC}') = 0.249$.

The gain (K) which must be contributed by the digital filter is:

$$K = \frac{K_F}{[M_P(\omega_{NC}')] [M_Z(\omega_{NC}')]}$$

$$= \frac{3.77}{[.53][.249]} = 28.57 \quad (54)$$

The values which should be programmed into the HCTL-1000 are therefore:

$$A = [256] [0.826] = 211.45 \cong 211(55)$$

$$B = [256] [0.9] = 230.4 \cong 230 \quad (56)$$

$$K = [4] [28.57] = 114.3 \cong 114 \quad (57)$$

Figure 27 shows the actual response of the closed loop system to a step input of 25 quadrature counts. (25 quadrature counts on a 256 count codewheel that is multiplied by 4 equals 8.79 de-

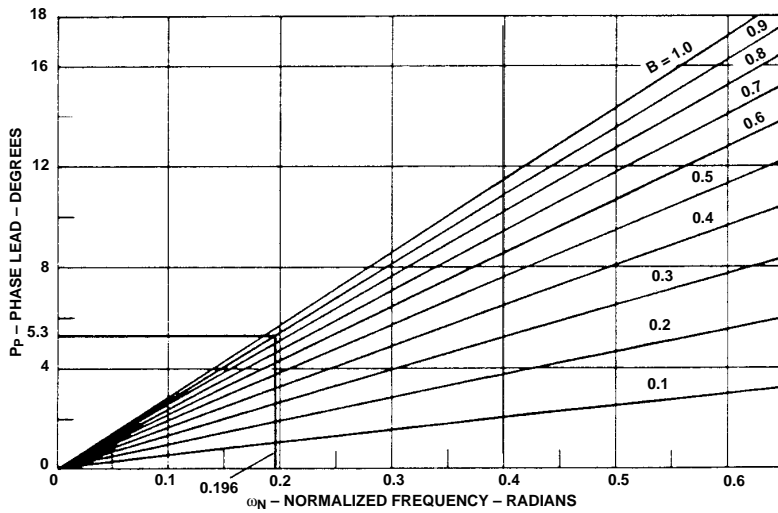


Figure 23. Phase Lead Contribution of the Pole Term to the Closed Loop System Compensator.

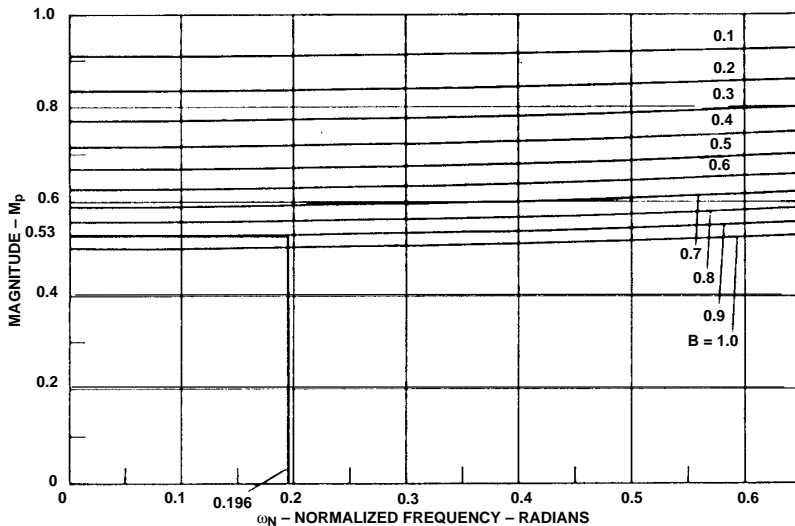


Figure 24. Magnitude Contribution of the Pole Term to the Closed Loop System Compensator.

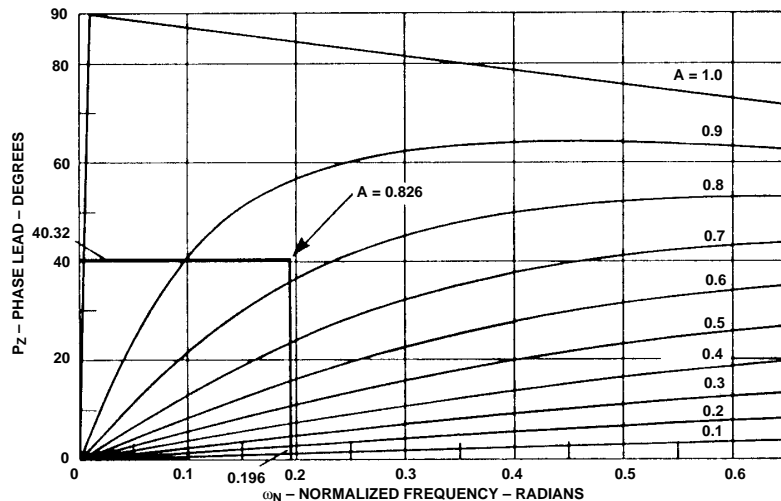


Figure 25. Phase Lead Contribution of the Zero Term to the Closed Loop System Compensator.

degrees of angular rotation). The amount of overshoot of the system's response is roughly consistent with a system with 40 degrees of phase margin.

For high phase margin systems the relationship between the risetime (t_R) and the bandwidth (BW) of a closed loop system can be approximated as:

$$t_R = \frac{2.2}{\text{BW (in radians/sec)}} \quad (58)$$

For low phase margin systems which have some overshoot in the closed loop step response, the relationship between the risetime (t_R) and the bandwidth (BW) of a closed loop system can be approximated as:

$$\frac{0.6}{\text{BW}} < t_R < \frac{2.2}{\text{BW}} \quad (59)$$

Figure 28 shows the actual risetime of the closed loop system to a step input of 25 quadrature counts. Note that the rise time is defined as the time required for the output to go from 10% to 90% of its final value. Since this system has some overshoot, equation (59) applies. The risetime (t_R) should therefore be:

$$\frac{0.6}{[60][6.28]} < t_R < \frac{2.2}{[60][6.28]} \text{ seconds}$$

.00159 < t_R < .0058 seconds
The risetime measured from figure 28 is:

$$t_R = 0.0050 \text{ seconds}$$

Therefore the measured risetime falls within the expected range of values for a bandwidth of 60 Hertz.

Appendix C: Symbols for Quantities and Their Units

Quantity	Definition	SI Units	Quantity	Definition	SI Units	
A	the HCTL-1000 digital compensation filter's programmable zero term	_____	M(s)	the open loop transfer function	varies	
B	the HCTL-1000 digital compensation filter's programmable pole term	_____	$M_U(\omega)$	the magnitude of the uncompensated open loop transfer function	_____	
BW	bandwidth	$\left[\begin{array}{l} \text{Hertz} \\ \text{radians/sec} \end{array} \right.$	N	number of slits in the encoder's codewheel	_____	
C	quadrature counts per revolution of the encoder's codewheel		counts	P_H	the phase margin of the uncompensated, open loop transfer function	degrees
D(z)	the transfer function of the HCTL-1000's digital filter		P_{HC}	desired, compensated open loop phase margin	degrees	
E	magnitude contribution of the encoder	counts/radian	$P_{HU}(\omega)$	the phase of the uncompensated, open loop transfer function	radians	
f_{CLK}	frequency of the HCTL-1000's external clock	hertz	P_L	the phase lead required from the digital filter	degrees	
G(s)	motor transfer function	varies	$P_M(\omega)$	phase contribution of the motor	radians	
G_M	the gain margin of the uncompensated open loop transfer function	decibels (db)	P_{MU}	the phase margin of the uncompensated, open loop system at the desired gain crossover frequency (ω_C')	degrees	
J	total system moment of inertia	kg-m ²	$P_P(\omega_N)$	the phase of the pole term at ω_{NC}'	degrees	
J_C	Moment of inertia of the encoder's codewheel	kg-m ²	$P_Z(\omega_N)$	the phase of the zero term at ω_{NC}	degrees	
J_L	Moment of inertia of the load	kg-m ²	$P_{ZOH}(\omega)$	phase of the zero order hold	radians	
J_M	Moment of inertia of the motor's armature	kg-m ²	R	terminal resistance of the motor	ohms	
K	the programmable gain of the HCTL-1000's digital compensation filter	_____	t	the sampling time of the HCTL-1000	seconds	
K_A	gain of the amplifier	$\left[\begin{array}{l} \text{volts/volt} \\ \text{amps/volt} \\ \text{volts/count} \\ \text{amps/count} \end{array} \right.$	T_E	electrical time constant of the motor	seconds	
K_D	gain of the DAC		volts/ count	T_M	mechanical time constant of the motor	seconds
K_E	motor voltage constant		volt-sec/rad	t_R	closed loop system rise time to a step response	seconds
K_F	the gain required from the HCTL-1000's digital filter to provide an overall gain equal to one at the desired gain crossover frequency (ω_C')	_____	ω	frequency	radians/sec	
K_{MC}	motor gain constant with a current source amp.	rad/amp-sec ²	ω_C	the open loop gain crossover frequency	radians/sec	
K_{MV}	motor gain constant with a voltage source amp.	rad/volt-sec	ω_C'	the desired open loop gain crossover frequency	radians/sec	
K_T	motor torque constant	N-m/amp	$\omega_N(\omega)$	normalized frequency	radians	
L	motor armature inductance	henries	ω_{NC}'	the normalized desired gain crossover frequency (ω_C')	radians	
$M_M(\omega)$	magnitude contribution of the motor	_____	ω_P	the open loop phase crossover frequency	radians/sec	
$M_P(\omega_N)$	the magnitude of the pole term at ω_{NC}'	_____	Z(s)	zero order hold transfer function	_____	
$M_Z(\omega_N)$	the magnitude of the zero term at ω_{NC}'	_____				

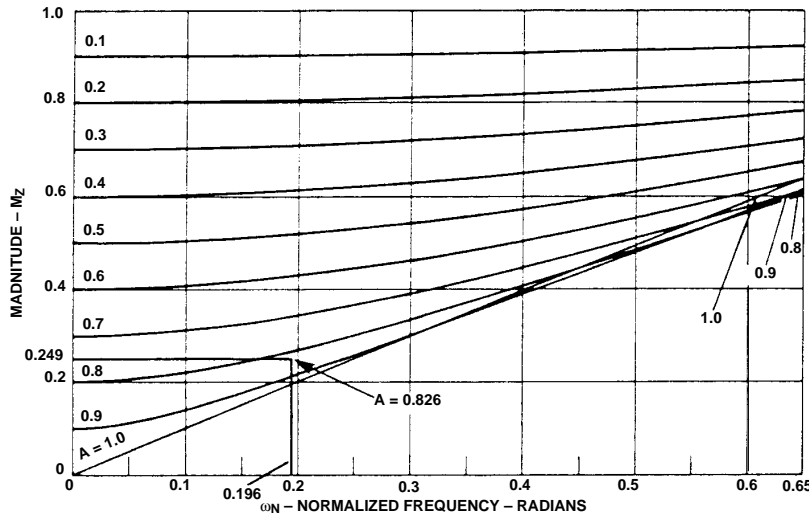


Figure 26. Magnitude Contribution of the Zero Term to the Closed Loop System Compensator.

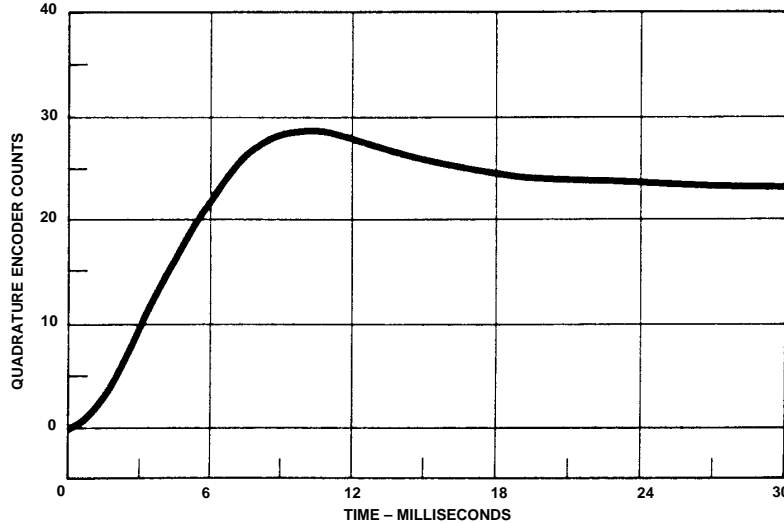


Figure 27. Response of the Closed Loop System to a Step Input of 25 Quadrature Encoder Counts.

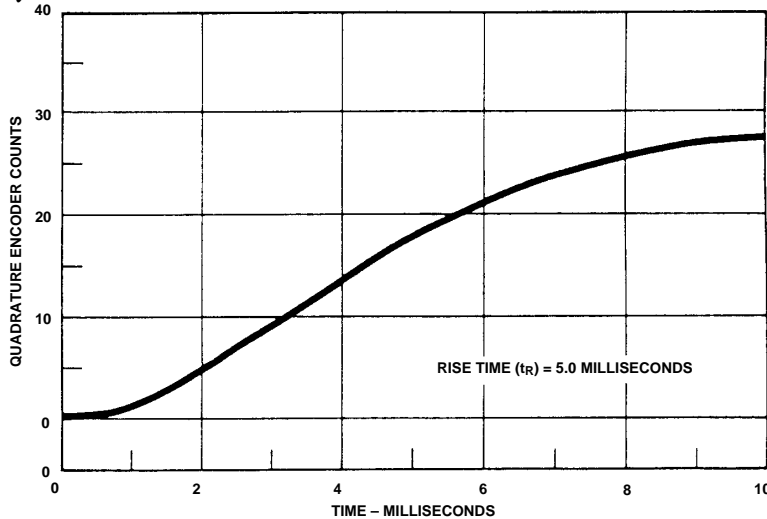


Figure 28. Rise Time of the Closed Loop System to a Step Input of 25 Quadrature Encoder Counts.

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