

Long-range superluminal pulse propagation in a coaxial photonic crystal

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We study the propagation of brief electric pulses along a coaxial line having a spatially periodic impedance. The periodicity causes anomalous dispersion and the appearance of a stop band in transmission near 10 MHz. Group velocities of up to three times the speed of light are observed in that spectral region, in accordance with calculations based on an effective index theory. © 2002 American Institute of Physics. [DOI: 10.1063/1.1432760]

Anomalous dispersion is known to allow group velocities for electromagnetic waves that exceed c , the speed of light in free space.¹ Such superluminal effects have been demonstrated in a number of optical experiments involving the tunneling of wave packets through resonant media^{2–5} and optical barriers.^{6–8} Important experiments have also been carried out in the microwave spectral region.^{9–12} Recently, special types of interfering Bessel beams were found to propagate at speeds slightly greater than c .¹³ A great deal of interest was generated in the scientific community over those results, as light was shed over the much-debated question of quantum and optical tunneling times. So far, however, superluminal effects have been either relatively modest in magnitude or limited to short range. In this letter, we report highly superluminal pulse propagation over distances much larger than ever observed before in a simple, wirelike structure—a coaxial line with periodic impedance, or coaxial photonic crystal.^{14,15}

Coaxial photonic crystals were recently introduced as macroscopic models for optical photonic crystals (PC's), to which they constitute an attractive alternative because of their macroscopic dimensions and added flexibility: the phase and amplitude of the electric field can be measured anywhere inside and outside the structure. Although they are limited to one dimension and void of polarization effects, their properties are also described by Maxwell's equations, and some linear¹⁴ and nonlinear¹⁵ properties similar to those encountered in optical PC's have been reported. The most notable feature of coaxial PC's is the existence of a forbidden gap of transmission at relatively low frequencies. In the optical case, stop bands are the result of reflections caused by an abrupt variation in the refractive index; in coaxial lines, they originate from an impedance mismatch. When the electric signal traverses an impedance boundary, it experiences a phase shift and a partial reflection that are calculated from the generalized optical Fresnel coefficients of reflection r and transmission t , namely $r = (z_i - z_t)/(z_i + z_t)$ and $t = 2z_i/(z_i + z_t)$, where z_i and z_t are the impedance of the incident and transmitted media. Thus, a periodic variation in the impedance of a medium can produce destructive interference for some wavelengths. As a result, the phase accumulated through the crystal changes rapidly with frequency, espe-

cially near the band gap, and anomalous dispersion and superluminal group velocity may occur.

The coaxial crystal used in this study consists of several unit cells, each made of two segments, one with 50 Ω (RG-58/U) and one with 75 Ω (RG-59/U) impedance. Each segment is characterized by the same phase velocity ($0.66c$) and length (5 m). As a result of impedance mismatch, 20% of the electric field is reflected at each interface. After assembling 12 unit cells in a row (120 m in total), the transmission spectrum plotted in Fig. 1 is obtained. A deep stop band occurs between 9 and 11 MHz, whereas outside the gap, transmission is mostly limited by the attenuation in the cables (25–35 dB/km).

To calculate the dispersive properties and expected group velocity of the structure, we use the effective index theory discussed in Refs. 16 and 17. The theory attributes the phase shift and scattering loss of the electric field through the crystal to an effective complex index of refraction. The real part of the index, or n_r , is obtained from the overall phase shift ϕ accumulated through the crystal of length D :

$$n_r = \frac{c\phi}{\omega D} \quad (1)$$

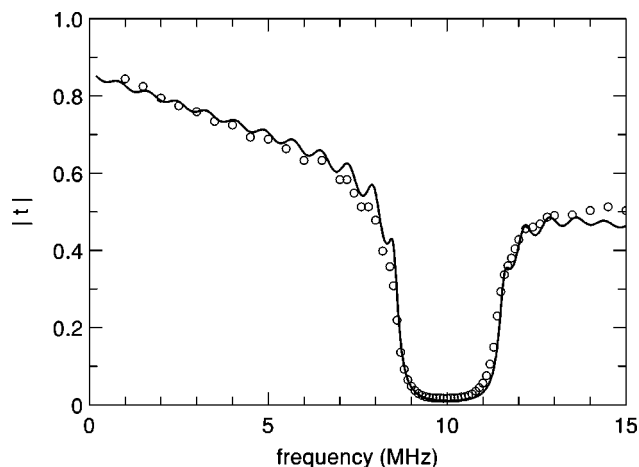


FIG. 1. Transmission coefficient of the electric field through a coaxial photonic crystal with 12 unit cells. The solid line is the expected transmission based on measured system parameters (no adjustments made).

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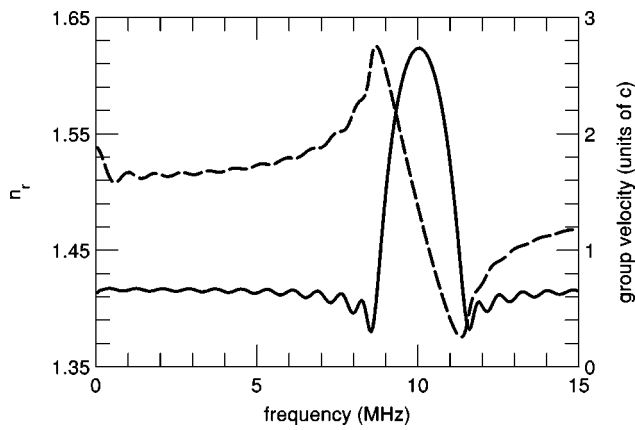


FIG. 2. Dispersion relation of the coaxial crystal calculated from the effective index theory. The dashed line is the real part of the effective refractive index and the solid line is the expected group velocity.

with

$$\phi = \arctan\left(\frac{\text{Im}(t)}{\text{Re}(t)}\right) + m\pi, \quad m=0,1,\dots, \quad (2)$$

where ω is the frequency, t is the complex coefficient of electric field transmission through the whole crystal, and m is unambiguously determined from the condition that $m=0$ as D or ω tend to zero. The group velocity is derived from n_r and is plotted in Fig. 2. Dispersion is fairly flat outside the band gap region, but the steep change in the effective refractive index predicts superluminal group velocities between 9 and 11 MHz.

The propagation velocity was determined by measuring the transit time of electric pulses along the coaxial crystal. A sinusoidal carrier wave with a Gaussian-shaped pulse envelope was launched in the structure using a programmable digital wave generator (HP 33120A). For carrier frequencies ranging from 5 to 15 MHz, the pulse duration was scaled from 6 to 2 μs , thereby keeping the number of cycles within the envelope constant at 30 while varying the bandwidth from 0.15 to 0.45 MHz. Figure 3 shows a typical trace for the emitted and transmitted pulses as measured with a dual

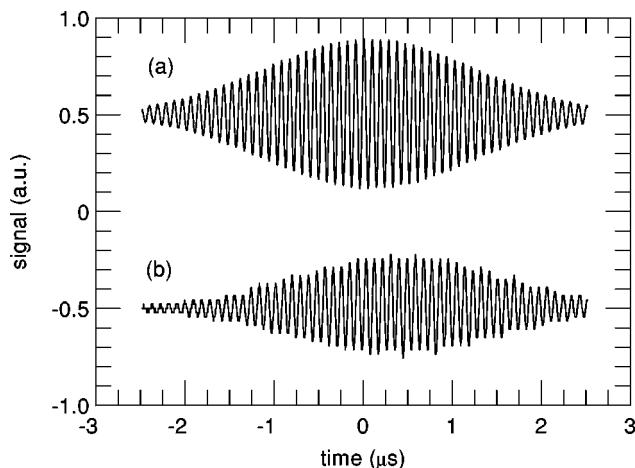


FIG. 3. Trace of a propagating pulse. Curve (a) is the emitted pulse at 10.8 MHz while curve (b) is the transmitted pulse (not plotted to scale vertically).

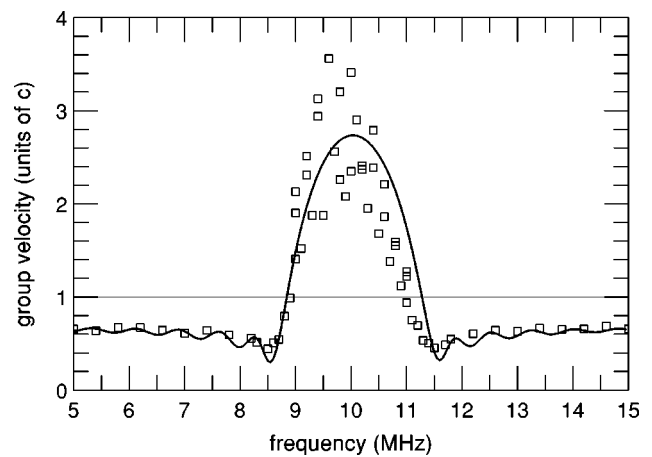


FIG. 4. Measured group velocity for Gaussian pulses travelling through the coaxial photonic crystal. The solid line is the theoretical calculation.

channel digital oscilloscope (Tektronix TDS 210). Because of noise, especially at frequencies inside the band gap, where transmission is weak, the transit time through the crystal was not determined from the absolute peak of each pulse but from their center of mass. This statistical-weighting approach gives a more accurate description of the pulse delay and is less vulnerable to local fluctuations. Several measurements were taken at each frequency, the results being plotted in Fig. 4. Outside the stop band, the group velocity remains subluminal and close to the phase velocity. However, between 9 and 11 MHz, it increases dramatically to between 2 and 3.5 c . The experimental error can be appreciated from the vertical dispersion of the data points, which tend to spread evenly around the theoretical curve. In the band gap region, where much of the signal is reflected back to the emitter, the uncertainty is of the order of $\pm 0.5 c$, but it is much smaller elsewhere ($\pm 0.05 c$). We should emphasize that there are other techniques available to determine the relative time delay, such as fitting the pulse to a Gaussian envelope, calculating its intensity, or even measuring the center of the pulse by eye, but they were found to give sensibly the same result.

As in the case of analogous optical experiments,^{6,7} this highly superluminal group velocity can be understood as an interference effect between spectral components of the pulse. In the highly dispersive band gap region, frequency components tend to interfere destructively in the tail of the pulse and be reflected backward. This effectively shifts the peak of the transmitted pulse forward in space. Although superluminal pulse propagation appears to violate the principle of causality, the issue is mostly a matter of definition. Group velocity, defined as the traveling speed of the peak or center of gravity of the pulse, can indeed exceed c or become negative.¹⁸ However, since the transmitted energy is at all times smaller than that of an equivalent pulse traveling through free space, no useful information can be sent at superluminal speed.^{19,20}

An interesting issue is the theoretical limitations to the transmission of superluminal pulses. As our results and Ref. 16 suggest, group velocity is inversely related to transmittance, so faster traveling speed is obtained at the expense of transmitted energy. On the other hand, multiple scattering of the electric field causes $|t|$ to scale as $\exp(-\alpha D)$, where α is

an effective attenuation coefficient related to scattering losses. The pulse transit time, in the absence of absorption (which tends to slow down the group velocity), is proportional to $D \exp(-\alpha D)$ and therefore vanishes for long crystal lengths. However, the pulse spectral width is a limitation whenever it becomes comparable to the photonic bandwidth of the crystal. Pulses that are too short in duration tend to be highly distorted at the exit, something we have observed both experimentally and with numerical simulations. This effect relates to the previous point: The ideal bit of information—a delta function, a very short pulse or a step function—is not expected to propagate at superluminal speeds in the crystal because of its excessive spectral width.

This very first demonstration of superluminal group velocity in a wirelike structure over distances exceeding 100 m opens the door to interesting possibilities. For example, highly accurate experiments to measure the tunneling time of wave packets through barriers could be carried out. From a technological point of view, coaxial lines still being at the basis of a large part of communications systems, faster pulse propagation could lead to improvements in the rate of transmission of information. The unique dispersion relation may allow for pulse propagation with minimal temporal broadening at some frequencies. Also, the signal velocity inside conventional, homogeneous coaxial cables is mostly limited by the phase velocity. By using cables with a modulated impedance and sending pulses near the photonic band edge, where the crystal is still fairly transparent, data bits speeds approaching c may be foreseeable.

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