Analytical Study of Transformer Inrush Current Transients and Its Applications

Sami G. Abdulsalam, *Student member, IEEE* and Wilsun Xu, *Fellow, IEEE*

Abstract-- This paper presents an improved design method for a novel transformer inrush current reduction scheme. The scheme energizes each phase of a transformer in sequence and uses a neutral resistor to limit the inrush current. Although experiment and simulation results have demonstrated the effectiveness of the scheme, the problem of how to select the neutral resistor for optimal performance has not been fully solved. In this paper, an analytical method that is based on the nonlinear circuit transient analysis is developed to solve this problem. The method models transformer nonlinearity using two linear circuits and derives a set of analytical equations for the waveform of the inrush current. In addition to establishing a set of formulas for optimal resistor determination, the results also reveal useful information regarding the inrush behavior of a transformer and the characteristics of the sequential energization scheme.

Keywords: Power Quality, Transformer, Inrush Current.

I. Introduction

INRUSH currents from transformer and reactor energization have always been a concern in power industry. Pre-insertion of series resistors and synchronous closing of circuit breakers are examples of the available mitigation techniques [1]-[3].

A neutral resistor based scheme for mitigating inrush currents was proposed by the authors in [4] and [5]. The scheme utilizes an optimally sized neutral resistor together with sequential energization each phase of the transformer. In [5], a design methodology for the neutral resistor size was developed based on steady state analysis. It was found that a neutral resistor size that is 8.5% of the un-saturated magnetizing reactance would lead 80% to 90% reductions on the inrush currents. However, the method did not analyze the resistor sizing issue from the perspective of switching transients due to technical difficulties.

Further study of the scheme revealed that a much lower resistor size could be equally effective. It was also found that the first phase energization leads to the highest inrush current among the three phases. If we can understand the transient characteristics of the first phase energization, it may be possible to refine the resistor sizing formula. With the help of nonlinear circuit theory [6], we managed to complete such analytical work. This paper will present the technique we used and the resultant findings.

The proposed method models transformer nonlinearity using two linear circuits presenting energized phase in saturated and un-saturated modes respectively. The significance of this work is that it is a rigorous analytical study of the transformer energization phenomenon. The results further reveal useful information regarding to the inrush behavior of transformers and the characteristics of the sequential energization scheme.

II. The Sequential Phase Energization Inrush Mitigation Scheme

The neutral resistor based inrush mitigation scheme shown in Fig. 1, adopts sequential phase energization together with an optimally sized neutral resistor, R_n. In view of the fact that the inrush currents are always unbalanced among three phases, a neutral resistor could provide some damping to the currents. This is the basis of the proposed idea. The idea was further improved by introducing delayed energization of each phase of the transformer. This improvement has made the proposed scheme almost as effective as the pre-insertion resistor scheme. The performance and characteristics of the method have been investigated using simulations and experiments in [4] and [5].

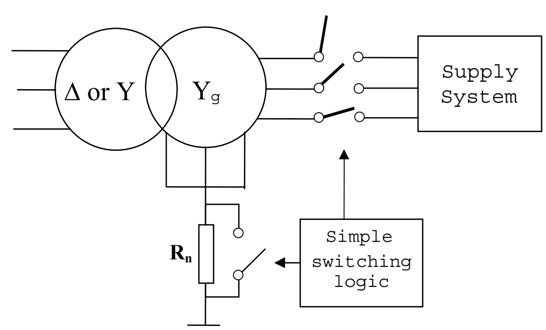

Fig. 1 The sequential phase energization inrush mitigation technique.

Since the scheme adopts sequential switching, each switching stage can be discussed separately. For first phase switching, the scheme performance is straightforward. The neutral resistor is in series with the energized phase and its effect will be similar to a pre-insertion resistor. When the third phase is energized, the voltage across the breaker to be closed is essentially zero due to the existence of delta secondary or

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three-legged core. So there are no switching transients for when the 3rd phase is energized [4] and [5].

The 2nd phase energization is the one most difficult to analyze. Fortunately, we discovered from numerous experimental and simulation studies that the inrush current produced from 2nd phase energization is smaller than that produced from 1st phase energization (when R_n is relatively small). This phenomenon is shown next and will be discussed in Section IV. The important conclusion at present is that the first phase energization should be the focus point for developing the optimal R_n formula. Experimental and simulation results of the I_{max}-R_n curves, representing the impact of R_n on the maximum inrush current of all phases, are shown in Fig. 2 and 3 respectively for a laboratory transformer 30kVA, 208/208, 3-limb, with Yg-Δ connection.

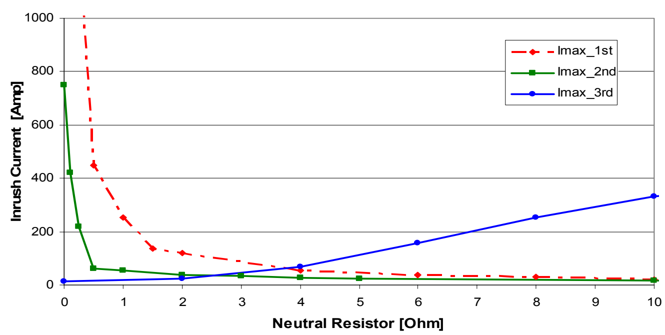


Fig. 2 Magnitude of inrush current as affected by the neutral resistor for a 30kVA, 208/208, Yg-Δ, 3 limb transformer. (Expiremental)

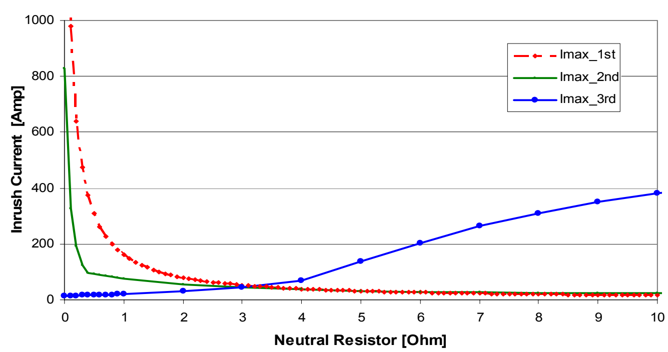


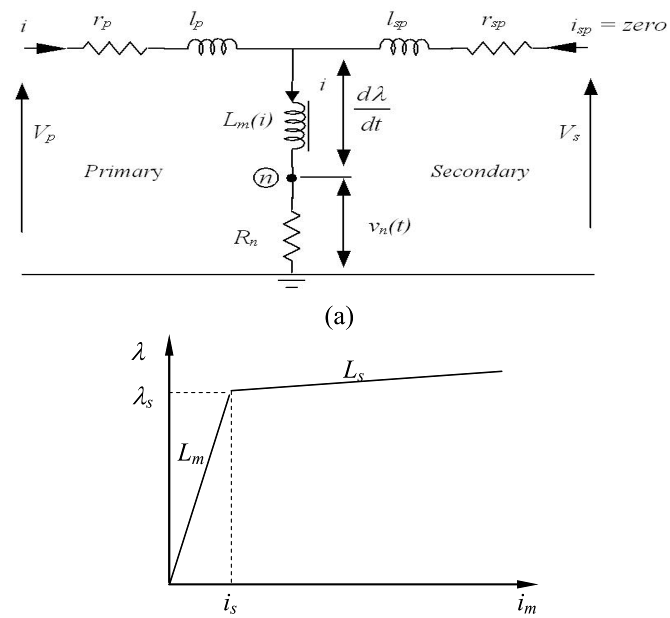
Fig. 3 Maximum inrush current as affected by the neutral resistor for a 30kVA, 208/208, Yg-D, 3 Limb transformer. (Simulation)

It can be seen that the maximum inrush current associated with the second phase energization is lower than that of the first phase energization for the same value of R_n. This is true for the region where the inrush current of the first phase is decreasing rapidly as R_n increases. As a result, we should focus on analyzing the first phase energization to develop a more precise selection method for the neutral resistor.

III. Analytical Expression for Inrush Current

An accurate analytical expression for inrush currents will lead to a solid design methodology for the neutral resistor size and more understanding of the scheme transient performance. The analytical expression will also eliminate the requirement of computer simulation for neutral resistor sizing on a case-by-case basis. Very few investigations in this field have been made and some formulas were given to predict the general wave shape, harmonic content or the maximum peak current [1], [6], [7], [8] and [9]. In most cases, the series impedance with the energized transformer 'resistive and reactive' has been neglected. For the presented application, it was required that the expression can accurately present the inrush current waveform taking into account system impedance, residual flux value and of course the neutral resistor itself.

The transformer behavior during first phase energization can be modeled through the simplified equivalent electric circuit shown in Fig. 4 together with an approximate two-slope saturation curve.



(a)

(b)

Fig. 4 (a) Transformer electrical equivalent circuit (per-phase) referred to the primary side. (b) Simplified, two sloped saturation curve.

As shown in Fig.4(a), r_p and l_p present primary resistance and leakage reactance. $L_m(i)$ represents the nonlinear inductance of the iron core as function of the magnetizing current. Secondary side resistance r_{sp} and leakage reactance l_{sp} as referred to primary side are also shown. V_p and V_s represent the primary and secondary phase to ground terminal voltages respectively. During first phase energization, the differential equation describing the behavior of the saturable iron core transformer can be written as follows;

$$v_p(t) = (r_p + R_n)\cdot i(t) + l_p \cdot \frac{di}{dt} + \frac{d\lambda}{dt}$$
$$v_p(t) = (r_p + R_n)\cdot i(t) + l_p \cdot \frac{di}{dt} + \frac{d\lambda}{di}\frac{di}{dt} \tag{4}$$

The rate of change of flux linkages with magnetizing current $d\lambda/di$ can be represented as an inductance equal to the slope of the λ-i curve. Eqn. (4) can be re-written as follows;

$$v_p(t) = (r_p + R_n)\cdot i(t) + l_p \cdot \frac{di}{dt} + L_{core}(\lambda)\cdot \frac{di}{dt} \tag{5}$$

The general solution of the differential equation (5) can be found through presenting the core nonlinear inductor in Fig.4.a as a linear inductor in un-saturated 'L_m' and saturated 'L_s' modes of operation, Fig 4.b.

Transformer performance during energization in unsaturated mode 'for each phase' will determine the time at which each phase will reach saturation first, depending on the switching angle and the amount of initial flux linkages λ_o. Generally, the initial 'or residual' flux will be below the saturation flux level and accordingly, the apparent magnetizing impedance will be very high compared to other linear elements in the series circuit. As a result, when the transformer is energized and λ_o is below λ_s, the total supply voltage will be mainly distributed across the magnetizing branch until saturation is reached. The saturation time 't_s' can be calculated as time required for the integral of the supply voltage added to the initial flux 'λ_o' to reach the saturation flux λ_s. Hysteresis effect 'usually presented as a resistance in parallel with the magnetizing reactance' will not affect estimation of the saturation time t_s.

$$\lambda_s = \int_0^{t_s} V_m \cdot sin(\omega \cdot t)dt + \lambda_o \tag{6}$$

$$t_s(\lambda_o) = \frac{1}{\omega}\cdot cos^{-1}(1-(\lambda_s - \lambda_o)/\lambda_n) \tag{7}$$

Where: λ_n nominal peak flux linkages.
ω angular frequency.
V_m nominal peak supply voltage.

After saturation is reached at $t=t_s$, the core inductance will be switched-in to equal the saturation inductance L_s with an initial saturation current i_s.

$$i(t) = \begin{cases} A_1 \cdot e^{-t/\tau_1} + B_1 \cdot sin(\omega \cdot t - \theta_1) & t \le t_s \\ (i_s + A_2)\cdot e^{-(t-t_s)/\tau_2} + B_2 \cdot sin(\omega \cdot t - \theta_2) & t > t_s \end{cases} \tag{8}$$

Where:

$$B_1 = \frac{V_m}{\sqrt{(r_p + R_n)^2 + (\omega\cdot(L_m + l_p))^2}} \qquad B_2 = \frac{V_m}{\sqrt{(r_p + R_n)^2 + (\omega\cdot(L_s + l_p))^2}}$$

$$A_1 = B_1 \cdot \sin(\theta_1) \qquad A_2 = B_2 \cdot \sin(\theta_2 - \omega \cdot t_s)$$

$$\theta_1 = \tan^{-1}\left(\frac{\omega\cdot(L_m + l_p)}{r_p + R_n}\right) \qquad \theta_2 = \tan^{-1}\left(\frac{\omega\cdot(L_s + l_p)}{r_p + R_n}\right)$$

$$i_s = i_s\big|_{\lambda_o = 0}\cdot(1 - \lambda_o/\lambda_s) \qquad \tau_1 = \frac{L_m + l_p}{r_p + R_n} \qquad \tau_2 = \frac{L_s + l_p}{r_p + R_n}$$

Figure 5 shows the first cycle, analytical and simulation waveform for the 30kVA transformer using neutral resistor values of 0.1, 0.5 and 1.0 [Ohm] respectively and a residual flux of 0.75 [p.u.]. Analytical and simulation results were obtained using the transformer data given in the appendix.

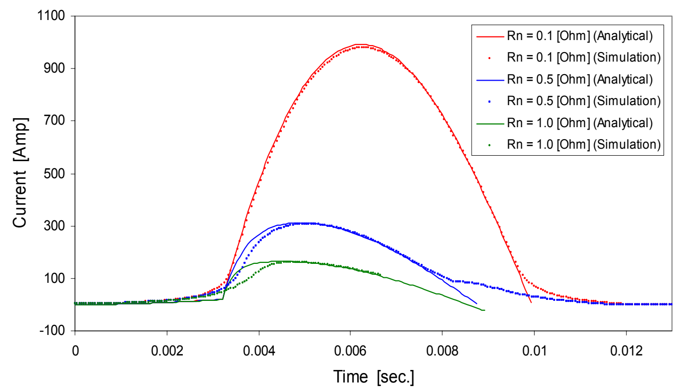


Fig. 5 Analytical and simulation inrush current waveforms (first cycle) for 30kVA Yg-Δ transformer.

Equation (8) can be further simplified to find the most severe inrush current peak as function of neutral resistor value during first phase switching. A switching angle of zero with a maximum residual flux of the same polarity as the applied sinusoidal will result in the maximum inrush current. The saturation current 'i_s' will be very small as compared to inrush current peak and can be neglected. It can also be assumed that the inrush peak value will exist during saturation when the sinusoidal term peaks. This assumption is valid since the time constant during saturation, $\tau_2(R_n)$, is small as R_n increases which will introduce a small shift in the peak current to appear slightly before the sinusoidal peak value. The peak time can be expressed as;

$$(\omega \cdot t_{peak}(R_n) - \theta_2(R_n)) = \pi/2$$

$$t_{peak}(R_n) = \frac{\pi/2 + \theta_2(R_n)}{\omega} \tag{9}$$

The simplified inrush current peak during first phase energization as function of R_n can be expressed as follows.

$$I_{peak}(R_n) = A_2 \cdot e^{-(t_{peak} - t_s)/\tau_2} + B_2 \tag{10}$$

Equation (10) was found to be very accurate as compared to simulation results. The $I_{peak}(R_n)$ 'analytical' and the I_{max}-R_n curves for the 30kVA lab transformer are shown in Fig. 6. It is clear that the $I_{peak}(R_n)$ equation can accurately determine the maximum inrush peak current for a given residual level and using only the simplified two slope saturation curve.

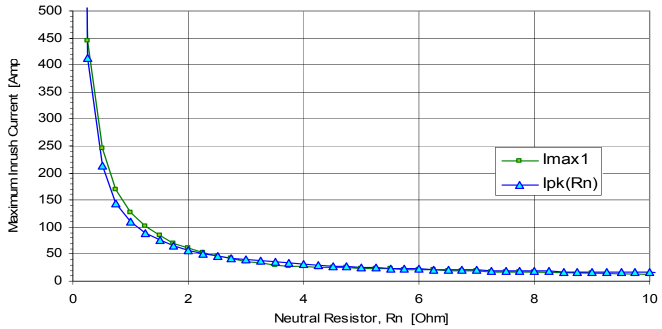


Fig. 6. $I_{peak}(R_n)$ compared to the simulation peak current for 30 kVA, 208/208 Yg-Δ, three limb transformer.

Sizing the neutral resistor based on Eqn. (10) and close to the knee of the $I_{max}(R_n)$ curve will insure a reduction of 80-90% of inrush current in all three phases as compared to the inrush magnitude with a solidly grounded connection, R_n=0.

IV. Second Phase Switching

Transformer behaviour during second phase switching was observed through simulation to vary with respect to connection and core structure type. Transformers with delta connected secondary or having multi limb structure have different behaviour during the second phase switching from that of single phase units without a delta winding. However, a general behaviour trend exists during the second switching stage for all transformer connections and core types for low neutral resistor values. In this section, the performance of the proposed inrush mitigation scheme during second stage switching will be discussed for small values of R_n.

A. Three Single Phase Units Connected in Yg-Y

For this condition, the transformer behavior can be modeled using two saturable inductor circuits representing each phase. The coupling between both switched phases is introduced only through the neutral resistor. For any energized phase j, the flux ϕ_j as function of the primary phase voltage v_{pj} and the neutral voltage v_n can be given by;

$$\phi_j = \int v_{pj}(t) \cdot dt - \int v_n(t) \cdot dt + \phi_{t=0} \qquad (11)$$

During second phase switching, the maximum inrush current can either exist on phase A or B. However, with phase A already in steady state, a disturbance in the flux equal to the difference between the rated and saturation flux values is required for phase A to reach saturation. For power transformers, the saturation flux is usually 1.25 p.u. of the rated flux or higher. Conservatively assuming that the reduction in flux in Phase B will result in an increase of the same amount of flux in phase A, it will be possible to increase R_n to achieve at least 25% reduction in its flux before phase A even reaches saturation. As R_n is increased further, more inrush current reduction can be achieved in phase B until both phases reach the same saturation level for a specific value of R_n. Actually, due to the phase difference in the supply voltage, the amount of disturbance in phase A flux will be less than the reduction in flux achievable in the switched phase B. Also, as the difference between the saturation and rated flux value increases, more reduction in phase B current can be achieved. The same conditions also apply during third switching stage.

B. Transformers with delta winding and/or 3-Limb structures

For transformers of this type, the performance during sequential switching will be quite different than the single phase Yg-Y transformers for the following reasons:

- Dynamic Flux will exist in un-energized phases.
- Inrush current can exist in one phase due to external saturation in un-energized phase (return path of the flux).

The existence of the dynamic flux will make the initial flux in the switched phase dependent on the instant of switching. It was found that the maximum inrush condition exists when switching at an angle of -30^o of the sinusoidal voltage waveform, which corresponds to zero initial flux in the switched phase B and in phase A at instant of switching. This finding clarifies that second phase I_{max}-R_n curve should be below the first switching curve for zero and small resistor values due to the absence of residual flux. With -30^o switching angle, the flux in phases A and B will be both positive and determined by the terminal voltage integral of both phases. This will lead phase C which represents the return path of both fluxes to saturate before any of the fluxes in phase A or B reach saturation values, Fig. 7.

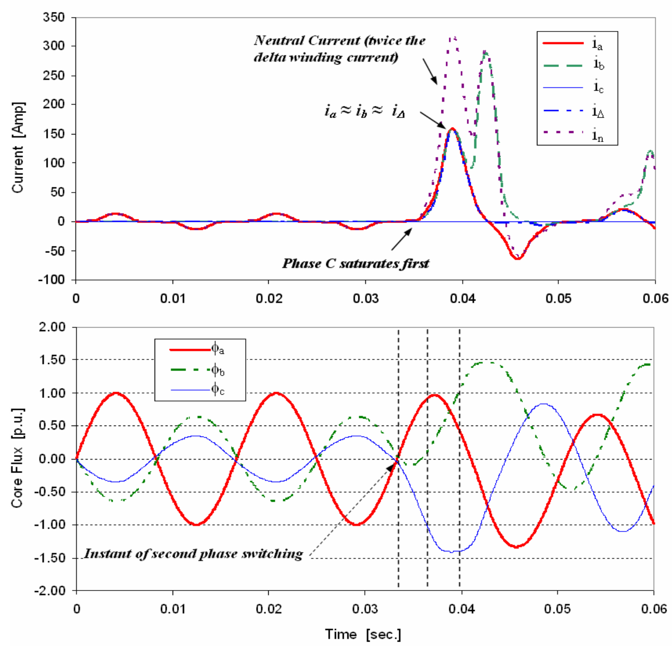


Fig. 7 Simulation of the 30kVA transformer during second phase switching condition showing the phase fluxes and effect of Delta winding current for small values of R_n = 0.1 [Ohm].

The saturation of phase C will drive a delta winding current equal to the magnetizing current of phase C under saturation.

As shown in Fig. 8, this current will be reflected as zero sequence current of the same magnitude flowing through phases A and B and a neutral current equal to twice the delta current. For phase B, both the integrals of the terminal and the neutral voltages have the same polarity and hence the delta winding will help reducing saturation level in phase B. For phase A, the supply voltage waveform will have opposite polarity to the neutral voltage, however, due to the difference between the saturation and rated flux values, the disturbance in phase a will be less than that observed in the switched phase B.

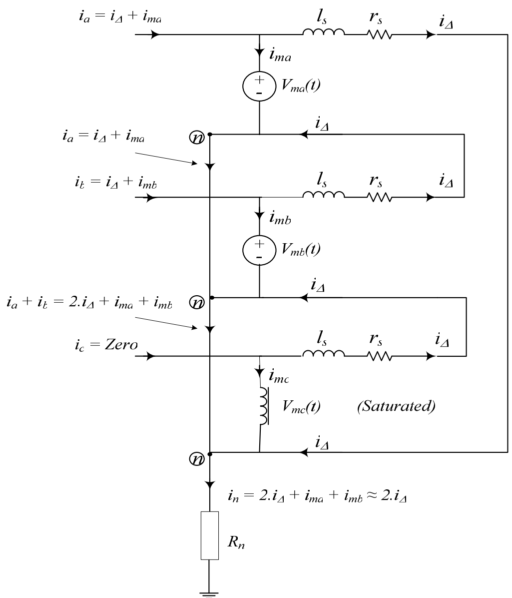


Fig. 8 Modeling the delta winding during saturation condition of phase C.

In case of delta winding absence in multi limb transformers, the behavior during second and third switching stages will depend on the number of core limbs. For 3-Limb transformers, the flux in the two energized limbs will add up into the third limb. As the third limb saturates, the return flux path of phase A and B will experience saturation and as a result a neutral current equals twice the phase current will flow. This will result in a similar effect to that from a delta winding. In the other hand, for transformers with 4 or 5 limbs the return path of the flux from phases A and B will always be un-saturated and the performance of the scheme will be similar to that of three single phase units connected in Yg-Y.

V. CONCLUSIONS

This paper presented an improved design methodology for a novel transformer inrush current reduction scheme. The main contributions are:

- An analytical methodology to analyze transformers during sequential energization has been presented. Effect of system impedance, neutral resistor and residual flux can also be taken into account.
- An accurate formula for the 1st phase maximum inrush current as function of neutral resistor value was derived.
- It was shown that the second phase switching condition could be analyzed considering separate nonlinear circuits for each energized phase, taking into account the core structure and the delta winding if it exists.

Experimental and simulation results revealed that the maximum inrush current magnitude due to 1st phase switching is always higher than that due to switching of the second and third phase. This finding made it possible to precisely size the neutral resistor based on the developed inrush current formula.

VI. APPENDIX

Laboratory transformer data:

208/208 [V], 30 [kVA], Yg-D 3-Limb transformer.

$r_p = 0.01$ [Ω], $l_p = 0.03291$ [mH], $\lambda_s = 1.4$ [p.u.],

$i_s = 45$ [amp], $L_s = 0.0807$ [mH], $N_p = 60$ turns.

System impedance:

$r_{system} = 0.12$ [Ω], $l_{system} = 0.12$ [mH].

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VIII. BIOGRAPHIES

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