sum. However, the definition of $\bar{P}(u_1,v_1)$, as well as the choice of the value of A, is *ad hoc*.

In the conception presented here, the state at the output of $\mathbf{U}_1$ and $\mathbf{U}_2$ is written as $|\Psi\rangle=\alpha_1|u_1,v_1\rangle+\alpha_2|u_1,v_2\rangle+\alpha_3|u_2,v_1\rangle+\alpha_4|u_2,v_2\rangle$, so that

$$\bar{P}(u_1,v_1)=|\alpha_1\alpha_4|^2-|\alpha_2\alpha_3|^2+A$$
$$=(|\alpha_1\alpha_4|-|\alpha_2\alpha_3|)(|\alpha_1\alpha_4|+|\alpha_2\alpha_3|)+A. \quad (10)$$

If we choose the phases of the elements of $\mathbf{U}_1$ and $\mathbf{U}_2$ such that $\sin\Phi=0$, where $\Phi=\theta_1+\theta_2+\varphi_1+\varphi_2$ (see Appendix), then both $\alpha_1\alpha_4$ and $\alpha_2\alpha_3$ are real positive quantities and $\bar{P}(u_1,v_1)=A\pm P_E(\alpha_1\alpha_4+\alpha_2\alpha_3)/2$; the quantity $\pm(\alpha_1\alpha_4+\alpha_2\alpha_3)/2$ fluctuates as the parameters of $\mathbf{U}_1$ and $\mathbf{U}_2$ are changed. The value of A should thus be chosen to be equal to the maximum absolute value of this latter quantity, which is 1/4 when $|\alpha_1|=|\alpha_4|$ and $|\alpha_2|=|\alpha_3|$. One can show that the choice of $\mathbf{U}_1$ and $\mathbf{U}_2$ that leads to the above condition is the same one that leads to the results provided in Refs. [12] and [13], which were related to interferometric complementarities but not to the degree of entanglement. The authors in Ref. [13] found that $V_{12}=2\kappa_1\kappa_2$, so that the measurement of two-particle visibility is tantamount to a measurement of the degree of entanglement P_E.

Note also that the visibilities of the singles rates (the one-particle visibilities) are all given by $\sqrt{1-P_E^2}$, so that in the context of our present construction, the complementarity of one- and two-particle visibilities [12,13] follows immediately from the normalization of the state vector.

Another interesting conclusion emerges from the following considerations. The state $|\Psi_e\rangle$ offers no *welcher-weg* (which-way) information about the two particles since each particle considered separately is in a maximally mixed state, whereas $|\Psi_f\rangle$ provides definite *welcher-weg* information about the two particles. Thus, the complementarity of one- and two-particle visibilities is the two-particle counterpart of the well-known complementarity for a single particle: that of *welcher-weg* information and interference visibility. In Ref. [13], the authors noted the similarity between these two complementarity relationships. The significance of this similarity is now clear.

We conclude that the proposed decomposition of Eq. (3) provides the underlying foundation for several seemingly different definitions of the degree of entanglement of a pure bipartite state of two qubits.

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APPENDIX: PROPERTIES OF THE DECOMPOSITION

Apply the most general local unitary transformation $\mathbf{U}=\mathbf{U}_1\otimes\mathbf{U}_2$ to the general bipartite state expressed in the Schmidt decomposition in Eq. (2):

$$\mathbf{U}_1=\begin{bmatrix} a_1 & -a_2 \\ a_2^* & a_1^* \end{bmatrix}, \quad \mathbf{U}_2=\begin{bmatrix} b_1 & -b_2 \\ b_2^* & b_1^* \end{bmatrix}, \quad (A1)$$

where $|a_1|^2+|a_2|^2=1$ and $|b_1|^2+|b_2|^2=1$; and $a_j=|a_j|e^{i\theta_j}$, $b_j=|b_j|e^{i\varphi_j}$, $j=1,2$; such that $|x_1\rangle\rightarrow a_1|u_1\rangle+a_2^*|u_2\rangle$, and so on. After transformation, the state in Eq. (2) may then be written as

$$|\Psi\rangle=\beta_1|u_1,v_1\rangle+\beta_2|u_1,v_2\rangle+\beta_3|u_2,v_1\rangle+\beta_4|u_2,v_2\rangle, \quad (A2)$$

where $\beta_1=\kappa_1a_1b_1+\kappa_2a_2b_2$, $\beta_2=\kappa_1a_1b_2^*-\kappa_2a_2b_1^*$, $\beta_3=\kappa_1a_2^*b_1-\kappa_2a_1^*b_2$, $\beta_4=\kappa_1a_2^*b_2^*+\kappa_2a_1^*b_1^*$. If we impose the conditions $\beta_3=0$ and $\beta_1=\beta_4$, we have $\kappa_2|a_1||b_2|=\kappa_1|a_2||b_1|$, $|a_1||b_1|=|a_2||b_2|$, $\theta_1+\varphi_1=\theta_2+\varphi_2$. Solving the first two relationships, we obtain $|a_1|=|b_2|=\sqrt{\kappa_1/(\kappa_1+\kappa_2)}$ and $|a_2|=|b_1|=\sqrt{\kappa_2/(\kappa_1+\kappa_2)}$; we then have $\beta_1=\beta_4=(p/\sqrt{2})e^{-i(\theta_1+\varphi_1)}$ and $\beta_2=\sqrt{1-p^2}e^{i(\theta_1-\varphi_2)}$, where $p^2=2\kappa_1\kappa_2$. Since the Schmidt coefficients are unique for any given state, the parameter p is also unique. We absorb the phases into the definition of $\mathbf{U}_1$ and $\mathbf{U}_2$ given in Eq. (A1) and thereby finally obtain the result given in Eq. (5). We can similarly impose the conditions $\beta_2=0$ and $\beta_1=\beta_4$ in Eq. (A2) to obtain the result given in Eq. (6). A similar analysis, but used for a different purpose, is the starting point of Ref. [14].

The parameter p may also be expressed in terms of the coefficients of $|\Psi\rangle$ in Eq. (1). A maximally entangled state takes the form $|\Psi_e\rangle=e^{i\gamma}(a_1|00\rangle+a_2|01\rangle-a_2^*|10\rangle+a_1^*|11\rangle)$, whereas a factorizable state takes the form $|\Psi_f\rangle=b_1|00\rangle+b_2|01\rangle+b_3|10\rangle+b_4|11\rangle$, where γ is a phase, $|a_1|^2+|a_2|^2=1/2$, and $b_1b_4-b_2b_3=0$. The coefficients of $|\Psi\rangle$ in Eq. (1) may be written in terms of the coefficients of $|\Psi_e\rangle$ and $|\Psi_f\rangle$, using Eq. (3), as $\alpha_1=pe^{i\gamma}a_1+\sqrt{1-p^2}e^{i\varphi}b_1$ and similarly for α_2, α_3, and α_4. It readily follows that

$$\alpha_1\alpha_4-\alpha_2\alpha_3=\frac{1}{2}p^2e^{i2\gamma}+p\sqrt{1-p^2}e^{i(\gamma+\varphi)}(a_1b_4+a_1^*b_1$$
$$-a_2b_3+a_2^*b_2). \quad (A3)$$

The expression in parentheses on the right-hand side of Eq. (A3) is precisely the orthogonality condition $\langle\Psi_e|\Psi_f\rangle=0$. It follows that $|\alpha_1\alpha_4-\alpha_2\alpha_3|=(1/2)p^2$, completing the proof of Eq. (8).