DM de maths :

$$f(x) = |x+1| + \frac{x}{x^2-1}$$

$D_f = \mathbb{R} \setminus \{-1;1\}$

1)

$$\forall x \in D_f, \quad |x+1| = \begin{cases} x+1 & \text{si } x > -1 \\ -x-1 & \text{si } x < -1 \end{cases}$$

a)

1er cas : si $x > -1$

$$f(x) = x+1+\frac{x}{x^2-1} = \frac{(x+1)(x+1)(x-1)+x}{x^2-1} = \frac{x^3+x^2-1}{x^2-1}$$

2e cas : si $x < -1$

$$f(x) = -x-1+\frac{x}{x^2-1} = \frac{(-x-1)(x^2-1)+x}{x^2-1} = \frac{-x^3-x^2+2x+1}{x^2-1}$$

b) $\forall x \in D_f$

- $\lim\limits_{x \to +\infty} f(x) = \lim \frac{x^3+x^2-1}{x^2-1} = +\infty$

- $\lim\limits_{x \to -\infty} f(x) = \lim \frac{-x^3-x^2+2x+1}{x^2-1} = +\infty$

- $\lim\limits_{x \to 1} f(x) = ?$

1er cas : $x > -1$

$$f(x) = \frac{x^3+x^2-1}{x^2-1}$$

Posons $f(x) = \frac{N}{D}$

$\lim\limits_{x \to 1} N = 1$; $\lim\limits_{x \to 1^\pm} D = 0^\pm$; $\lim\limits_{x \to 1^\pm} f(x) = \pm\infty$

- $\lim\limits_{x \to -1} f(x) = ?$

2e cas : $x < -1$

$$f(x) = \frac{-x^3-x^2+2x+1}{x^2-1}$$

Posons $f(x) = \frac{N}{D}$

$\lim\limits_{x \to -1} N = -1$; $\lim\limits_{x \to -1^\pm} D = 0^\mp$; $\lim\limits_{x \to -1^\pm} f(x) = \mp\infty$