1. Non

$$\Gamma(z) = \int_0^{+\infty} x^{z-1} e^{-x} dx$$

converge si $x > 0$, diverge pour $x \leq 0$.

2. $q \in \mathbb{R}_+, n \in \mathbb{N}$

$$\begin{aligned}\binom{-\frac{1}{q}}{n} &= \frac{\Gamma\left(\frac{-1+q}{q}\right)}{\Gamma(1+n)\,\Gamma\left(\frac{-1+(-n+1)q}{q}\right)} \\ &= \frac{\left(-\frac{1}{q}\right)!}{n!\left(-\left(\frac{1}{q}+n\right)\right)!}\end{aligned}$$

On a

$$\begin{aligned}\Gamma(z)\,\Gamma(1-z) &= \frac{\pi}{\sin(\pi z)} \\ \Gamma(z+1) &= z\Gamma(z) \\ \Gamma(1/2) &= \sqrt{\pi}\end{aligned}$$

ou

$$(-z)! = \frac{\pi}{(z-1)!\sin(\pi z)} \text{ si } z \in \mathbb{N}$$

$$\begin{aligned}\binom{-\frac{1}{q}}{n} &= \frac{\Gamma\left(\frac{-1+q}{q}\right)}{\Gamma(1+n)\,\Gamma\left(\frac{-1+(-n+1)q}{q}\right)} \\ &= \frac{\frac{\pi}{\left(\frac{1}{q}-1\right)!\sin\left(\frac{\pi}{q}\right)}}{n!\frac{\pi}{\left(\frac{1}{q}+n-1\right)!\sin\left(\pi\left(\frac{1}{q}+n\right)\right)}} \\ &= \frac{\left(\frac{1}{q}+n-1\right)!\sin\left(\pi\left(\frac{1}{q}+n\right)\right)}{n!\left(\frac{1}{q}-1\right)!\sin\left(\frac{\pi}{q}\right)}\end{aligned}$$

3. Exemple:

$$\begin{aligned}\binom{-\frac{1}{2}}{2} &= \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma(3)\,\Gamma\left(-\frac{3}{2}\right)} \\ &= \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma(3)\,\Gamma\left(-\frac{3}{2}\right)} \\ &= \frac{\sqrt{\pi}}{2(4/3)\sqrt{\pi}} \\ &= \frac{3}{8}\end{aligned}$$

$$
\begin{aligned}
\Gamma\left(-\frac{3}{2}\right) &= \frac{\pi}{\Gamma\left(1+3/2\right)\sin\left(-\pi 3/2\right)} \\
&= \frac{\pi}{\Gamma\left(5/2\right)} \\
&= \frac{\pi}{\Gamma\left(1+3/2\right) = 3/2\Gamma\left(3/2\right) = 3/2\Gamma\left(1+1/2\right) = 3/2 * 1/2\Gamma\left(1/2\right) = 3/4\sqrt{\pi}} \\
&= (4/3)\sqrt{\pi}
\end{aligned}
$$