**1D Richard's equation :**

$$\frac{\partial\theta}{\partial\psi}\frac{\partial\psi}{\partial t} - \frac{\partial}{\partial z}\left(K(\psi)\frac{\partial\psi}{\partial z}\right) + \frac{\partial}{\partial z}K(\psi) = S_r \tag{1}$$

$$\psi(L,t) = \beta(t) \tag{2}$$

$$K(\psi) - K(\psi)\frac{\partial\psi}{\partial z} = q(t) \text{ at } z = 0 \tag{3}$$

$$\psi(z,0) = \psi_0(z). \tag{4}$$

To solve equation (1), the soil water retention function is introduced to eliminate one of the two dependent variables.

$$\theta(\psi) = \theta_r + \frac{\alpha(\theta_s - \theta_r)}{\alpha + |\psi|^\beta}$$

The unsaturated conductivity $K$ is given by

$$K(\psi) = \frac{\alpha}{\alpha + |\psi - z|^\gamma}K_s$$

What you need to do is to solve the following transformed equation

$$\frac{\partial\tilde{\theta}}{\partial t} = \frac{\partial^2\phi}{\partial z^2} + S_r,\ \bar{\theta}(\phi) = \theta(h) \tag{5}$$

$$\frac{\partial\phi}{\partial z} = -q(t) \text{ at } z = 0 \tag{6}$$

$$\phi(L,t) = \bar{\beta}(t) \tag{7}$$

$$\phi(z,0) = \phi_0(z). \tag{8}$$

Thus, the unknown of the equation is $\phi$. Start discretizing the PDE

$$\frac{\partial\tilde{\theta}(\phi)}{\partial t} = \frac{\partial^2\phi}{\partial z^2} + S$$

$$\frac{\bar{\theta}_i^{n+1} - \bar{\theta}_i^n}{\Delta t} = \frac{\phi_{i+1}^{t+1} - 2\phi_i^{t+1} + \phi_{i-1}^{t+1}}{(\Delta z)^2} + S_i$$

where $\bar{\theta}_i^n = \bar{\theta}(\phi_i^n) = \theta(h_i^n)$.

**The key idea is to solve it for $\boldsymbol{\phi_i^{n+1}}$ using the same code as lab14a.f90 which requires evaluating $\boldsymbol{\bar{\theta}(\phi)}$ using the relations**

$$\bar{\theta}(\phi) = \theta(h) = \theta_r + \frac{\alpha(\theta_s - \theta_r)}{\alpha + |h - z|^\beta}, \quad \phi(h) = \int_0^h \bar{K}(\lambda)\,d\lambda, \quad \bar{K}(\lambda) = \frac{\alpha}{\alpha + |\lambda|^\gamma}K_s$$