

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

Et

$$y''(x) = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

On pose :

$$E(x) = y''(x) - xy(x)$$

$$E(x) = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^{n+1}$$

Ou

$$E(x) = \sum_{n=0}^{\infty} n(n-1)a_{n+2}x^n + \sum_{n=0}^{\infty} a_{n-1}x^n$$

Finalement on a :

$$E(x) = \sum_{n=0}^{\infty} [n(n-1)a_{n+2} - a_{n-1}]x^n$$

Comme $E(x)=0$, on a $n(n-1)a_{n+2} - a_{n-1} = 0$