1 Intertemporal Utility Maximization

We assume that an individual lives from period 1 to period T and retirement occurs at time R. Utility is derived from consumption and is assumed to be isoelastic and exhibits constant relative risk aversion (CRRA). The consumer maximizes lifetime utility (C_τ) :

$$U(C_\tau) = \sum_{\tau=1}^{T} (1+\rho)^{1-\tau} \frac{C_\tau^{1-\gamma}}{1-\gamma}$$

s.t.

$$\sum_{\tau=1}^{T} C_\tau (1+r)^{1-\tau} = \sum_{\tau=1}^{T} y_\tau (1+r)^{1-\tau} = \sum_{\tau=1}^{R} E_\tau (1+r)^{1-\tau} + \sum_{\tau=R+1}^{T} B_\tau (1+r)^{1-\tau}$$

The first-order condition and the budget constraint define the optimal consumption path :

$$C_\tau = C_1 \left(\left(\frac{1+r}{1+\rho} \right)^{\frac{1}{\gamma}} \right)^{\tau-1}, \tau = 2, ..., T$$

where

$$C_1 = \left(\sum_{\tau=1}^{T} \lambda^{\tau-1} \right)^{-1} \left(\sum_{\tau=1}^{T} (1+r)^{1-\tau} y_\tau \right)$$

After writing the Lagrangian, what are the (mathematical) steps to come by these results ?

For those who prefer the continuous version of this model please see Gale, *The effects of Pensions on Households wealth : A reevaluation of theory and evidence*, 1998.

Thanks a lot !