

EXERCICE 2

$$1) y_{m+1} = y_m + \frac{h}{2} (y'(t_m) + y'(t_m) + h y'(t_m))$$

$$\Leftrightarrow y_m + \frac{h}{2} (-y(t_m) - y(t_m) + h y'(t_m))$$

$$\Leftrightarrow y_m - \frac{h}{2} y(t_m) (h+2)$$

$$\Leftrightarrow y_m \left(-\frac{h^2}{2} - h + 1 \right)$$

$$\underline{y_{m+1} \Leftrightarrow y_0 \left(-\frac{h^2}{2} - h + 1 \right)^{m+1}}$$

$$2) y_{m+1} = S(h) y_0 = y_0 \left(-\frac{h^2}{2} - h + 1 \right)^{m+1}$$

$$\Leftrightarrow \underline{S(h) = \left(-\frac{h^2}{2} - h + 1 \right)^{m+1}}$$

$$3) |S(h)| < 1 \quad \text{d } h > 0$$

$$\left| \left(-\frac{h^2}{2} - h + 1 \right)^{m+1} \right| < 1$$

$$\left| -\frac{h^2}{2} - h + 1 \right| < 1$$

$$-1 < -\frac{h^2}{2} - h + 1 < 1$$

$$0 < -\frac{h^2}{2} - h + 2 < 2$$

$$\Rightarrow \begin{cases} -\frac{h^2}{2} - h + 2 > 0 & \textcircled{1} \\ -\frac{h^2}{2} - h + 2 < 2 & \textcircled{2} \\ h > 0 & \textcircled{3} \end{cases}$$