Exo 1 3)

$x^2 y'' + y = (\ln x)^2$

On pose $t = \ln x$

$z(t) = z(\ln x) = y(x)$

$y'(x) = z'(t) = z'(\ln x) = \frac{1}{x} z'(\ln x) = \frac{1}{e^t} z'(t)$

$y''(x) = (z'(t))' = \left(\frac{1}{x} z'(\ln x)\right)' = -\frac{1}{x^2} z' \ln(x) + \frac{1}{x^2} z'' \ln(x)$

$= \frac{1}{e^{2t}} \left(z''(t) - z'(t)\right)$

Donc $\boxed{x^2 y'' + y = \ln(x)^2 = \cancel{e^{2t}} \times \frac{1}{\cancel{e^{2t}}} z''(t) - z'(t) + z(t) = t^2}$

$z''(t) - z'(t) + z(t) = t^2$

Solution homogène : $z_H = z'' - z' + z = 0 \rightarrow r^2 - r + 1 = 0 \Rightarrow b^2 - 4ac = (1)^2 - 4 \times 1 \times 1 = -3$

$\Delta = (\sqrt{3}\ i)^2$

2 racines complexes : $r_1 = \frac{1}{2} + i \frac{\sqrt{3}}{2}$

$r_2 = \frac{1}{2} - i \frac{\sqrt{3}}{2}$

$\boxed{z_H = e^{\frac{1}{2}} + \cos\left(\frac{\sqrt{3}}{2} t\right)(C_1 + C_2)}$

Solution particulière par identification de polynôme.

$z_p = at^2 + bt + c$

$z'_p = 2at + b$

$z''_p = 2a$

$\Rightarrow at^2 + (-2a - b)t + (2a - b + c) = t^2$

$$\begin{cases} a = 1 \\ -2a + b = 0 \\ 2a - b + c = 0 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 2 \\ c = 0 \end{cases}$$

Donc $\boxed{z_p = t^2 + 2t}$

$z(t) = z_H + z_p = e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2} t\right)(C_1 + C_2) + t^2 + 2t$

$\rightarrow \boxed{y(x) = \sqrt{x} \cos\left(\frac{\sqrt{3}}{2} \ln x\right)(C_1 + C_2) + (\ln x)^2 + 2\ln x}$

C_1 et C_2 en fonction des conditions initiales.