

$$\begin{cases} \frac{\partial u}{\partial t} = a_1 * \frac{\partial^2 u}{\partial x^2} + a_2 * \frac{\partial u}{\partial x} + a_3 * u * e^{-\frac{k_1}{v}} \\ \frac{\partial v}{\partial t} = b_1 * \frac{\partial^2 v}{\partial x^2} + b_2 * \frac{\partial v}{\partial x} + b_3 * v * e^{-\frac{k_1}{v}} \end{cases}$$

Initials conditions :

$$\begin{cases} u(x, 0) = 0 \\ v(x, 0) = k_2 \end{cases}$$

Boundaries conditions:

$$\begin{cases} u(0, t) = 0 \quad \text{and} \quad \left(\frac{\partial u}{\partial x}\right)(x_L, t) = 0 \\ v(0, t) = k_3 \quad \text{and} \quad \left(\frac{\partial v}{\partial x}\right)(x_L, t) = 0 \end{cases}$$

$$\text{Let } u = \begin{pmatrix} u_1 \\ \vdots \\ u_{N+1} \end{pmatrix} \text{ and } v = \begin{pmatrix} v_1 \\ \vdots \\ v_{N+1} \end{pmatrix}$$

$$\text{Let } U = \begin{pmatrix} u \\ v \end{pmatrix} \in \mathbb{R}^{2*(N+2)}$$

$$\text{Then } \frac{\partial U}{\partial t} = \begin{pmatrix} a_1 * \frac{\partial^2 u}{\partial x^2} + a_2 * \frac{\partial u}{\partial x} + a_3 * u * e^{-\frac{k_1}{v}} \\ b_1 * \frac{\partial^2 v}{\partial x^2} + b_2 * \frac{\partial v}{\partial x} + b_3 * v * e^{-\frac{k_1}{v}} \end{pmatrix}$$

$$\text{We know that : } \left(\frac{\partial y}{\partial x}\right)_i \approx \frac{y_{i+1} - y_i}{\Delta x} \text{ and } \left(\frac{\partial^2 y}{\partial x^2}\right)_i \approx \frac{y_{i+1} - 2 * y_i + y_{i-1}}{\Delta x}$$

*So, for $i = 2 : N$ and for $i = (N + 3):(2 * N + 1)$, we end up with :*

$$\frac{dU}{dt} = \begin{pmatrix} a_1 * \frac{u_{i+1} - 2 * u_i + u_{i-1}}{\Delta x} + a_2 * \frac{u_{i+1} - u_i}{\Delta x} + a_3 * u_i * e^{-\frac{k_1}{v_i}} \\ b_1 * \frac{v_{i+1} - 2 * v_i + v_{i-1}}{\Delta x} + b_2 * \frac{v_{i+1} - v_i}{\Delta x} + b_3 * v_i * e^{-\frac{k_1}{v_i}} \end{pmatrix}$$

for $i = 1, N + 2$, the initials conditions already give us the value of U

We can also approximate the boundaries conditions for the 1st derivative:

$$\left(\frac{\partial y}{\partial x}\right)(x_L, t) = 0 \leftrightarrow \frac{y_{N+1} - y_N}{\Delta x} = 0 \leftrightarrow y_{N+1} = y_N$$

Then for $i = N + 1, 2 * N + 2$, we get :

$$U_{N+1} = U_N \text{ and } U_{2*N+2} = U_{2*N+1}$$