Question 1 :

a.) $f(0) = 1 + \frac{1}{3} \rightsquigarrow \frac{4}{3} \quad \Rightarrow (0; \frac{4}{3})$

$f(1) = 0 \quad \Rightarrow (1; 0)$

$f(2) = \frac{2}{3} \quad \Rightarrow (2; \frac{2}{3})$

$$f'(0) = \frac{\frac{1}{3} - \frac{4}{3}}{\frac{1}{3} - 0} = \frac{-\frac{3}{3}}{\frac{1}{3}} = -\frac{1}{1/3} = -3$$

$\hookrightarrow (0; \frac{4}{3})$ et $(\frac{1}{3}; \frac{1}{3})$
$\quad x_1 \; y_1 \quad x_2 \; y_2$

$$f'(1) = \frac{0-0}{1-1} = 0$$

$\hookrightarrow (1; 0)$ et $(1; 0)$
$\quad x_1 \; y_1 \quad x_2 \; y_2$

$$f'(2) = \frac{1 - \frac{2}{3}}{\frac{7}{3} - 2} = \frac{1/3}{1/3} = 1$$

$\hookrightarrow (2; \frac{2}{3})$ et $(\frac{7}{3}; 1)$

b.) $t \equiv y = f'(a)(x-a) + f(a)$

- pour $x = 0$:

$\hookrightarrow f(a) = f(0) = \frac{4}{3} \quad \hookrightarrow f'(a) = f'(0) = -3$

$\Rightarrow (-3).(x-0) + \frac{4}{3}$

$t_0 \equiv y = -3x + \frac{4}{3}$

- pour $x = 1$:

$\hookrightarrow f(a) = f(1) = 0 \quad \hookrightarrow f'(a) = f'(1) = 0$

$t_1 \equiv y = 0$

↘

- pour $x = 2$:

$\hookrightarrow f(a) = f(2) = \frac{2}{3}$ $\quad$ $\hookrightarrow f'(a) = f'(2) = 1$

$\Rightarrow 1.(x-2) + \frac{2}{3}$

$\Rightarrow x - 2 + \frac{2}{3}$

$t_2 \equiv y = x - \frac{4}{3}$

Question 2 :

a.) $|z_1| = |2i - 1| = \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

b.) $\overline{z_1} - 2z_2 = \overline{2i-1} - 2.(4+i) = 2i + 1 - 8 - 2i = -7$

c.) $z_1 . z_3 = (2i-1)(3-4i)$

$= 6i - 8i^2 - 3 + 4i$

$= 10i - 8i^2 - 3$ $\quad$ $\rightsquigarrow 8i^2 = 8.(-1) = -8$

$= 10i + 8 - 3$

$= 10i + 5$

d.) $\frac{z_1}{z_2} = \frac{2i-1}{4+i} = \frac{(2i-1)(4-i)}{(4+i)(4-i)} = \frac{8i - 2i^2 - 4 + i}{16 - i^2}$

$= \frac{8i - 2.(-1) - 4 + i}{16 - (-1)} = \frac{-4 + i + 8i + 2}{16+1}$

$= \frac{-2 + 9i}{17}$ $\rightsquigarrow \frac{-2}{17} + \frac{9}{17} i$

Question 3 :

a.) $2 \operatorname{cis} 255°$

$r = 2 \;;\; \theta = 225°$

$\cos 225 = \cos(180+45) = -\cos 45° = \dfrac{-\sqrt{2}}{2}$

$\sin 225 = \text{''} = \dfrac{-\sqrt{2}}{2}$

- formule : $r(\cos\theta + i\sin\theta)$

$\Rightarrow 2.(\cos 225° + i\sin 225°)$

$\Rightarrow 2.\left(-\dfrac{\sqrt{2}}{2} + \left(-\dfrac{\sqrt{2}}{2}\right)i\right)$

$\Rightarrow -\sqrt{2} - \sqrt{2}\,i$

b.) $-3\left(\operatorname{cis}\dfrac{\pi}{2} - \operatorname{cis}(-\pi)\right)$

- pour $\dfrac{\pi}{2}$: $\operatorname{cis}\dfrac{\pi}{2} = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}$

$\cos\dfrac{\pi}{2} = 0$ et $\sin\dfrac{\pi}{2} = 1$

$\Rightarrow \operatorname{cis}\dfrac{\pi}{2} = 0 + i \cdot 1 = i$ $\rightsquigarrow (\cos + i\sin)$

- pour $-\pi$: $\cos(-\pi) = -1$ et $\sin(-\pi) = 0$

$\Rightarrow \operatorname{cis}(-\pi) = -1 + i.0 = -1$

remplacer dans la formule :

$\hookrightarrow -3(i - (-1)$

$\Rightarrow -3(i+1)$

$\Rightarrow -3i - 3$

Question 4 :

a.) -2 ; $r = |z|$

$r = |-2|$

$r = \sqrt{(-2)^2}$

$r = \sqrt{4} = 2$.

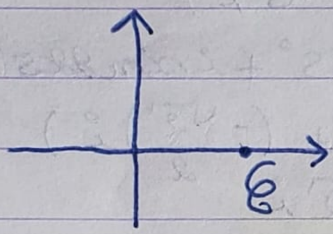
$\sin \frac{b}{r} = \frac{0}{2} = 0$ *regarder tableau trigono.

$\cos \frac{a}{r} = \frac{-2}{2} = -1$

$\hookrightarrow \pi = 180°$

$\Rightarrow 2 \text{ cis } 180°$.

- plan de Gauss :



?

b.) $-\sqrt{3} + 3i$

$r = |z|$

$r = |-\sqrt{3} + 3i|$

$r = \sqrt{(-\sqrt{3})^2 + 3^2}$

$r = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$

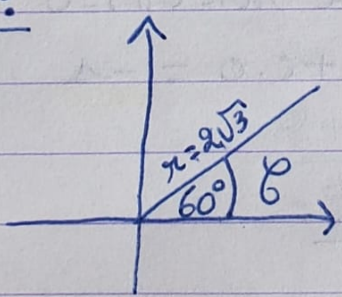
$\sin \frac{b}{r} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\cos \frac{a}{r} = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2} = \frac{\pi}{3}$

$\hookrightarrow \frac{\pi}{3} = 60°$

$\Rightarrow 2\sqrt{3} \text{ cis } 60°$.

- plan de Gauss :



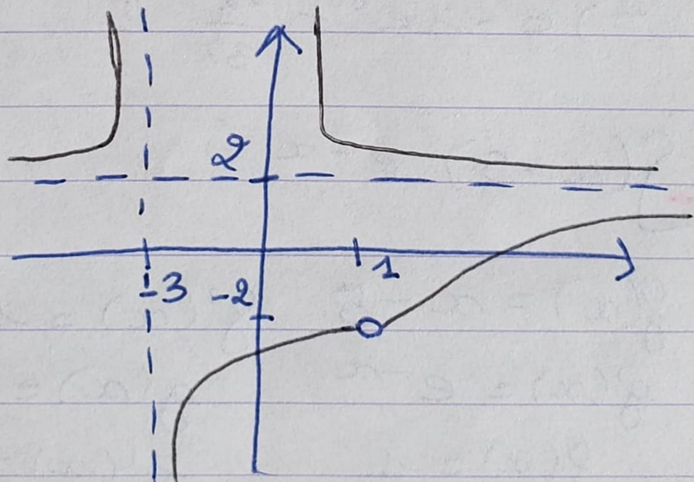
Question 5:

$\lim_{x \to -\infty} f(x) \nexists$

$\lim_{x \to +\infty} f(x) = 2 \Rightarrow AH \equiv y = 2.$

$\left.\begin{array}{l} \lim_{x \to -3^+} f(x) = -\infty \\ \lim_{x \to -3^-} f(x) = +\infty \end{array}\right\} AV \equiv x = -3$

$\lim_{x \to 1} f(x) = -2 \Rightarrow$ pt creux en $(1; -2)$.



f.) $f(x) = (e^{2023})'$
$= e^{2023}$

g.) $f(x) = (\arcsin x)^5$
$= 5.(\arcsin x)^4$
$= 5.(\arcsin)^4 . \frac{1}{\sqrt{1-x^2}}$

h.) $f(u) = (e^{u^3-7u+3})'$
$= (e^{u^3-7u+3})' . (u^3-7u+3)'$
$= e^{u^3-7u+3} . 3u^2 - 7$

Question 6 :

a.) $\lim_{x \to -3} \frac{x^2+6x+9}{x+3} = \frac{(-3)^2+6.(-3)+9}{(-3)+3} = \frac{0}{0}$

$\hookrightarrow \left(\frac{x^2+6x+9}{x+3}\right)' = \frac{2x+6}{1} = \frac{2.(-3)+6}{1} = 0$

$\Rightarrow$ point creux en $(-3;0)$

b.) $\lim_{x \to -\infty} \sqrt{-x^2+x^4-2} = \sqrt{x^4} = \sqrt{\infty^4} = +\infty$

$\Rightarrow$ pas d'AH donc AO.

↘

Question 7:

a.) $f(x) = (4x^5 - 3x^2 + 103)'$

$= 4.5x^4 - 2.3x$

$= 20x^4 - 6x$

b.) $f(x) = \left(\frac{1}{3x^6}\right)'$

$= \frac{(3x^6)'}{(3x^6)^2} = \frac{18x^5}{3x^{12}}$

c.) $f(z) = (\arctan(z) \cdot \cos(z))'$

$= (\arctan(z))' \cdot \cos(z) - \arctan(z) \cdot (\cos(z))'$

$= \frac{1}{1+z^2} \cdot \cos(z) - \arctan(z) \cdot -\sin(z)$

$= \frac{\cos(z)}{1+z^2} - \arctan(z) \cdot -\sin(z)$

d.) $f(t) = (2^t - \log_2(t) + x)'$

$= 2^t \ln 2 - \frac{1}{t \ln_2} + 1$

e.) $f(x) = \left(\frac{4x^3}{x^2+1}\right)' \rightsquigarrow \frac{f' \cdot g - f \cdot g'}{g^2}$

$= \frac{(4x^3)' \cdot (x^2+1) - 4x^3 \cdot (x^2+1)'}{(x^2+1)^2}$

$= \frac{12x^4 + 12x^2 - 8x^3}{(x^2+1)^2}$

c) $\lim\limits_{x \to 3} \left(\sqrt{x-2} - \sqrt{2-x}\right) = \left(\sqrt{3-2} - \sqrt{2-3}\right)$

$= \sqrt{1} - \sqrt{1} = 0.$

↳ point creux en $(3;0)$.

d) $\lim\limits_{x \to +\infty} \dfrac{-5x^4+x-1}{2x^4+x^3-x+1} = \dfrac{-5x^4}{2x^4} = \dfrac{\infty}{\infty}.$

↳ $\dfrac{-5x^4}{2x^4} = \dfrac{-5}{2}$ donc $AH \equiv x = \dfrac{-5}{2}.$

e) $\lim\limits_{x \to -2} \dfrac{x+5}{x-2} = \dfrac{(-2)+5}{(-2)-2} = \dfrac{3}{-4}.$

↳ point creux en $\left(-2; \dfrac{-3}{4}\right).$

f) $\lim\limits_{x \to -\infty} \left(\sqrt{4x^2-3x+1} + 2x\right) = \sqrt{4x^2} = -\infty$

↳ donc pas d'AH

g) $\lim\limits_{x \to 1} \dfrac{2x-1}{x^2-1} = \dfrac{2 \cdot 1 - 1}{1^2-1} = \dfrac{1}{0} \rightsquigarrow \pm\infty?$

		-1		1	
x^2-1	+	0	+	0	+

↳ $AV \equiv x = 1.$

Question 8 :

superficie □ doit être 1Km²:

$$x \cdot y = 1$$

=> x : longueur du côté adjacent (km).

=> y : longueur de l'autre côte (km).

Coût total C de la clôture :

=> long de la route : 500€/km.

=> long des 3 autres : 300€/km. $(x+2y)$.

$$C = 500x + 300(x+2y)$$
$$= 800x + 600y$$

pour $y = \frac{1}{x}$ ⇝ $C = 800x + 600\left(\frac{1}{x}\right)$.

Trouver dérivée de C :

$$\frac{dC}{dx} = 800 - 600 \cdot \frac{1}{x^2} \Rightarrow 800 - 600 \cdot \frac{1}{x^2} = 0$$

$$\Rightarrow 800 = 600 \cdot \frac{1}{x^2} \Rightarrow 800x^2 = 600$$

$$\Rightarrow x^2 = \frac{600}{800} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Trouver y en utilisant x :

$$y = \frac{1}{x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Vérifier minimum :

$$S ⇝ 600 \cdot \frac{2}{x^3}$$

$$⇝ 600 \cdot \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^3} > 0$$

Les dimensions qui minimisent le coût totale de la clôture sont : $\boxed{\begin{aligned} x &= \sqrt{3}/2 \text{ Km} \\ y &= 2\sqrt{3}/3 \text{ Km} \end{aligned}}$

Question 9 :

a.) $\int x^5 - \frac{1}{x^2} + \frac{1}{x} + 7 \, dx$

$\Rightarrow \frac{x^6}{6} - \frac{1}{x} + \ln|x| + 7x + c$

Q9:

b.) $\int (10-6x^2)\sqrt{x^3-5x}\,dx$

$\Rightarrow -2\int(-5+3x^2)(x^3-5x)^{1/2}dx$

$\frac{du}{dx} = 3x^2-5$

$du = 3x^2-5\,dx$

$= -2\int u^{1/2}du = -2 \cdot \frac{u^{3/2}}{3/2}$

$= -2u^{3/2} \cdot \frac{2}{3}$

$= \frac{-4\sqrt{(x^3-5x)}^3}{3} = \frac{4x^3-20x \cdot \sqrt{x^3-5x}}{3}$

c.) $\int_1^8 e^u - \frac{1}{3\sqrt{u}}\,du \rightsquigarrow \left[e^u - \frac{1}{3\sqrt{u}}\right]_1^8$

$\Rightarrow \left(e^1 - \frac{1}{3\sqrt{1}}\right) - \left(e^8 - \frac{1}{3\sqrt{8}}\right)$

$\Rightarrow e - 1e^8 + \frac{\sqrt{2}}{32}$?

d.) $\int (x-5).e^{-x}$.

$f(x) = x-5 \quad f'(x) = 1$

$g'(x) = e^{-x} \quad g(x) = -e^{-x}$.

$f(x) = 1 \quad f'(x) = 0$

$g'(x) = -e^{-x} \quad g(x) = -e^{x}$

$\Rightarrow (x-5).e^{-x} - \int 1.-e^{-x}\,dx$

$\Rightarrow (x-5).e^{-x} + \int e^{-x}\,dx$

$\Rightarrow (x-5)e^{-x} - e^{-x}$?

Question 10 :

$f'(x) = \frac{3}{\cos^2 x} - \sin x$ et $f(\pi) = 1$.

$\hookrightarrow 3 \int \frac{1}{\cos^2 x} dx - \int \sin x \, dx$

$= 3 \tan x + \cos x + k$