I need to calculate the variation of :

$$S(\bar{\xi}) = \int_{0}^{\bar{\xi}} \left\| x^{1}_{,\xi}(\xi) \right\| d\xi \tag{1}$$

First choice :

$$\frac{\partial S(\bar{\xi})}{\partial \xi} = \left\| x^{P}_{,\bar{\xi}} \right\| \frac{\partial \bar{\xi}}{\partial \xi} \tag{2}$$

Witch gives :

$$\delta S(\bar{\xi}) = \left\| x^{P}_{,\bar{\xi}} \right\| \delta \bar{\xi}$$

I have applied Leibniz formula :

$$\frac{d}{dx} \int_{u(x)}^{v(x)} f(t) dt = f(u(x)) \frac{du(x)}{dx} - f(v(x)) \frac{dv(x)}{dx}$$

Second choice

$$\delta S(\bar{\xi}) = \int_{0}^{\bar{\xi}(x_i^A, x_i^s)} \frac{\partial}{\partial x_i^A} \left\| x^{1}_{,\xi}(x_i^A, \xi) \right\| d\xi \delta x_i^A + \left\| x^{P}_{,\bar{\xi}} \right\| \delta \bar{\xi} = \int_{0}^{\bar{\xi}(x_i^A, x_i^s)} \frac{x^{1}_{,\xi}}{\left\| x^{1}_{,\xi} \right\|} N^{A}_{,\xi} d\xi \delta x_i^A + \left\| x^{P}_{,\bar{\xi}} \right\| \delta \bar{\xi}$$

with :

$$x^{1}_{,\xi} = N^{A}_{,\xi} x_i^A$$

I don't Know witch of this tow choice was wright ?

Thanks for Your help !