

$$g_{\theta\theta} = r^2 \quad g_{\phi\phi} = r^2 \sin^2 \theta \quad r \text{ est un paramètre}$$

Calcul direct en coordonnées rectangulaires

$$\Gamma_{\phi\phi}^{\theta} = -\frac{1}{2g_{\theta\theta}} g_{\phi\phi,\theta} \quad \text{notation dérivée } \partial g / \partial_x = g_{,x}$$

$$\Gamma_{\phi\phi}^{\theta} = -\sin \theta \cos \theta$$

$$\Gamma_{\theta\phi}^{\phi} = \Gamma_{\phi\theta}^{\phi} = \frac{1}{2g_{\phi\phi}} g_{\phi\phi,\theta} = \cot \theta \quad \text{pas de signe -}$$

Formule directe du tenseur de Ricci

$$R_{\mu\nu} = \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\beta\alpha}^{\alpha} \Gamma_{\mu\nu}^{\beta} - \Gamma_{\beta\nu}^{\alpha} \Gamma_{\mu\alpha}^{\beta}$$

2 composantes non nulles

$$R_{\theta\theta} = -\Gamma_{\theta\phi,\theta}^{\phi} - \Gamma_{\phi\theta}^{\phi} \Gamma_{\theta\phi}^{\phi} = \frac{1}{\sin^2 \theta} - \cot^2 \theta = 1$$

$$R_{\phi\phi} = \Gamma_{\phi\phi,\theta}^{\theta} + \Gamma_{\theta\phi}^{\phi} \Gamma_{\phi\phi}^{\theta} - \left(\Gamma_{\theta\phi}^{\phi} \Gamma_{\phi\phi}^{\theta} + \Gamma_{\phi\phi}^{\theta} \Gamma_{\phi\theta}^{\phi} \right)$$

$$R_{\phi\phi} = \left(\sin^2 \theta - \cos^2 \theta \right) - \cos^2 \theta + 2 \cos^2 \theta = \sin^2 \theta$$

Pour le tenseur mixte :

$$R_{\theta}^{\theta} = R_{\theta\theta} / g_{\theta\theta} = \frac{1}{r^2}$$

$$R_{\phi}^{\phi} = R_{\phi\phi} / g_{\phi\phi} = \frac{1}{r^2}$$