The electrostatic energy is given by :

$$U = \sum_{i<j} \frac{q_i \cdot q_j}{4\pi\epsilon_0 |\vec{r_i} - \vec{r_j}|} + \sum_{i<j} \frac{Q_i \cdot Q_j}{4\pi\epsilon_0 |\vec{R_i} - \vec{R_j}|} + \sum_{i,j} \frac{q_i \cdot Q_j}{4\pi\epsilon_0 |\vec{r_i} - \vec{R_j}|} \tag{1}$$

The "mechanic" energy is given by :

$$U_M = \sum_{i<j} U_{ij}(\vec{R_i} - \vec{R_j}) \tag{2}$$

If N-1 free particles are already at their final positions, the work done to brought the Nth charge is :

$$W_N = \int_{\vec{r_N}}^{\infty} q_N \cdot \left(\sum_{i=1}^{N-1} \frac{q_i \cdot (\vec{r} - \vec{r_i}(\vec{r}))}{4\pi\epsilon_0 |\vec{r} - \vec{r_i}(\vec{r})|^3} + \sum_{m} \frac{Q_j \cdot (\vec{r} - \vec{R_j}(\vec{r}))}{4\pi\epsilon_0 |\vec{r} - \vec{R_j}(\vec{r})|^3} \right) \cdot d\vec{r} \tag{3}$$

Where the positions of the bounded charges are functions of the positions of the Nth free charge. The total work is given by :
$W_{total} = \sum_N W_N$

The net force applied on jth bounded particle is zero :

$$\vec{F_j} = 0 = \sum_{k \neq j} \frac{Q_j \cdot Q_k \cdot (\vec{R_k}(\vec{r}) - \vec{R_j}(\vec{r}))}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r})|^3} + \sum_{i=1}^{N-1} \frac{q_i \cdot Q_j \cdot (\vec{R_j}(\vec{r}) - \vec{r_i})}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{r_i}|^3} + \frac{q_N \cdot Q_j \cdot (\vec{R_j}(\vec{r}) - \vec{r})}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{r}|^3} + \sum_{k \neq mj} -\vec{\nabla}.U_{jk}(\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r})) \tag{4}$$

We can rewrite this equation :

$$-\frac{q_N \cdot Q_j \cdot (\vec{R_j}(\vec{r}) - \vec{r})}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{r}|^3} = \sum_{k \neq j} \frac{Q_j \cdot Q_k \cdot (\vec{R_k}(\vec{r}) - \vec{R_j}(\vec{r}))}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r})|^3} + \sum_{i=1}^{N-1} \frac{q_i \cdot Q_j \cdot (\vec{R_j}(\vec{r}) - \vec{r_i})}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{r_i}|^3} + \sum_{k \neq mj} -\vec{\nabla}.U_{jk}(\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r})) \tag{5}$$

Let's replace it in the expression of the work :

$$W_N = \int_{\vec{r_N}}^{\infty} \left(\sum_{j=1}^{N-1} \frac{q_N \cdot q_i \cdot (\vec{r} - \vec{r_i})}{4\pi\epsilon_0 |\vec{r} - \vec{r_i}|^3} - \sum_{j} \left(\sum_{k \neq j} \frac{Q_j \cdot Q_k \cdot (\vec{R_k}(\vec{r}) - \vec{R_j}(\vec{r}))}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r})|^3} + \sum_{i=1}^{N-1} \frac{q_i \cdot Q_j \cdot (\vec{R_j}(\vec{r}) - \vec{r_i})}{4\pi\epsilon_0 |\vec{R_j}(\vec{r}) - \vec{r_i}|^3} + \sum_{k \neq j} -\vec{\nabla}.U_{jk}(\vec{R_j}(\vec{r}) - \vec{R_k}(\vec{r}))) \right) \right) \cdot d\vec{r} \tag{6}$$

Let's now perform the integration.
$\int_{\vec{r_N}}^{\infty} \frac{\vec{r} - \vec{r_i}}{|\vec{r} - \vec{r_i}|^3} \cdot d\vec{r} = \int_{\vec{r_N}}^{\infty} \frac{|\vec{r} - \vec{r_i}| \cdot cos(\theta_i)}{|\vec{r} - \vec{r_i}|^3} dr = \int_{\vec{r_N}}^{\infty} \frac{cos(\theta_i)}{|\vec{r} - \vec{r_i}|^2} dr = \left[\frac{1}{|\vec{r} - \vec{r_i}|} \right]_{\vec{r_N}}^{\infty} = \frac{1}{|\vec{r_N} - \vec{r_i}|}$
where θ_i is the angle between $\vec{r} - \vec{r_i}$ and $\vec{r}$.
$W_N = q_N \cdot \left(\sum_{i=1}^{N-1} \frac{q_N \cdot q_i}{4\pi\epsilon_0 |\vec{r_N} - \vec{r_i}|} - \sum_{j} \left(\sum_{k \neq j} \frac{Q_j \cdot Q_k}{4\pi\epsilon_0 |\vec{R_j}(\vec{r_N}) - \vec{R_k}(\vec{r_N})|} + \sum_{i=1}^{N-1} \frac{q_i \cdot Q_j}{4\pi\epsilon_0 |\vec{R_j}(\vec{r_N}) - \vec{r_i}|} + \sum_{k \neq j} U_{jk}(\vec{R_j}(\vec{r_N}) - \vec{R_k}(\vec{r_N}))) \right) \right)$