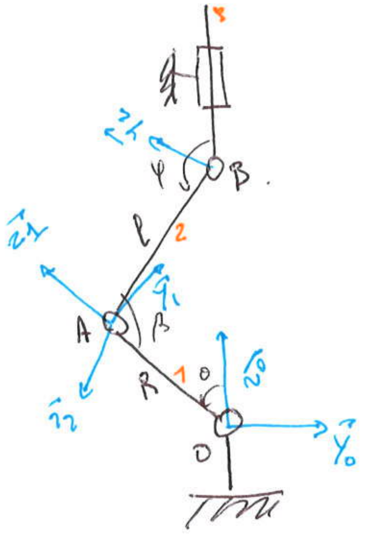
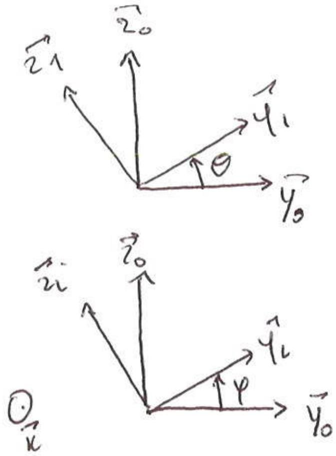
Objectif

Déterminer les efforts dans la liaison bielle manivelle sans prise en compte de la combustion.

Schéma cinématique et repères





Détermination du champ de vitesse de la bielle

$$\vec{\Omega}(2/0) = \vec{\Omega}(2/1) + \vec{\Omega}(1/0)$$

$$\vec{\Omega}(1/0) = \dot{\theta}\,\vec{x}$$

$$\beta = 180 - \theta - (180 - \varphi).$$
$$\beta = \pi - \theta - \pi + \varphi$$
$$= -\theta + \varphi.$$

$$\vec{\Omega}(2/0) = (\dot{\varphi} - \dot{\theta})\,\vec{x}.$$

$\vec{\Omega}(2/0) = \dot{\varphi}\,\vec{x}$ (Que l'on avait eu plus simplement en passant par (3))

Or $R\sin\theta = \ell\sin(\pi - \varphi) = \ell\sin\varphi$. et $\varphi = \pi - \mathrm{Arcsin}\left(\frac{R}{\ell}\sin\theta\right)$.

$$\cos\varphi = -\frac{1}{\ell}\sqrt{L^2 - R^2\sin^2\theta}$$

Ainsi $R\dot{\theta}\cos\theta = \ell\dot{\varphi}\cos\varphi.$

$$R\dot{\theta}\cos\theta = \ell\dot{\varphi} \times \left(\frac{-1}{\ell}\right)\sqrt{\ell^2 - R^2\sin^2\theta}$$

$$\dot{\varphi} = -\frac{R\dot{\theta}\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}$$

et donc $\boxed{\vec{\Omega}(2/0) = -\frac{R\dot{\theta}\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}\,\vec{x}}$

Et $\boxed{\vec{V}_A(2/0) = \vec{V}_A(1/0) = -R\dot{\theta}\,\vec{y_1}.}$

Détermination du torseur cinétique de la bielle en A

$$\vec{\sigma}_A(2/0) = m_B\, \overrightarrow{AG} \wedge \vec{V}_A(2/0) + \underline{\underline{I_G}} \cdot \vec{\Omega}(2/0).$$

Notons $\overrightarrow{BG} = l_1\, \vec{z}_2$ $\qquad m_B$: Masse bielle.

$$\underline{\underline{I_G}} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}_{(\vec{x}_2, \vec{y}_2, \vec{z}_2)}$$

Ainsi $\vec{\sigma}_A(2/0) = m_B\, (l - l_1)\, (-\vec{z}_2) \wedge (-R\dot{\theta})\, \vec{y}_1 + \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \begin{pmatrix} \Omega(2/0) \\ 0 \\ 0 \end{pmatrix}.$

$$\vec{y}_1 = \cos\theta\, \vec{y}_0 + \sin\theta\, \vec{z}_0$$
$$= \cos\theta \left(\cos\varphi\, \vec{y}_2 - \sin\varphi\, \vec{z}_2\right) + \sin\theta \left(\cos\varphi\, \vec{z}_2 + \sin\varphi\, \vec{y}_2\right).$$

$$\vec{y}_1 = \begin{pmatrix} 0 \\ \cos\theta\cos\varphi + \sin\theta\sin\varphi \\ \sin\theta\cos\varphi - \cos\theta\sin\varphi \end{pmatrix}$$

$$\sigma_A(2/0) = -m_B (l - l_1) R\dot{\theta} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ \cos\theta\cos\varphi + \sin\theta\sin\varphi \\ \sin\theta\cos\varphi - \cos\theta\sin\varphi \end{pmatrix} + I_{xx}\, \Omega(2/0)\, \vec{x}.$$

$$\boxed{\sigma_A(2/0) = \left[- m_B (l - l_1) R\dot{\theta} \left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right) + I_{xx}\, \Omega(2/0)\right] \vec{x}}$$

$$\text{et } \cos\varphi = -\frac{1}{l}\sqrt{l^2 - R^2\sin^2\theta} \qquad \sin\varphi = \frac{R}{l}\sin\theta$$

(laissé sous cette forme pour ne pas alourdir).

Détermination de $V_G(2/0)$.

$$\vec{V}_G(2/0) = \vec{V}_A(2/0) + \vec{\Omega}(2/0) \wedge \overrightarrow{AG}$$
$$= -R\dot{\theta}\, \vec{y}_1 + \vec{\Omega}(2/0) \wedge (L - L_1)\, (-\vec{z}_2).$$
$$= -R\dot{\theta}\, \vec{y}_1 + \Omega(2/0)(L_1 - L) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$
$$= -R\dot{\theta}\, \vec{y}_1 + (L - L_1)\, \Omega(2/0)\, \vec{y}_2$$
$$= -R\dot{\theta}\left(\cos\theta\, \vec{y}_0 + \sin\theta\, \vec{z}_0\right) + (L - L_1)\, \Omega(2/0) \left(\cos\varphi\, \vec{y}_0 + \sin\varphi\, \vec{z}_0\right).$$

$$\boxed{\vec{V}_G(2/0) = \left[(L - L_1)\, \Omega(2/0) \cos\varphi - R\dot{\theta}\cos\theta\right] \vec{y}_0 + \left[(L - L_1)\, \Omega(2/0) \sin\varphi - R\dot{\theta}\sin\theta\right] \vec{z}_0}$$

Détermination du moment dynamique de la bielle en A .. m_B

$$\vec{\delta}_A(2/0) = \frac{d}{dt}\left(\vec{\sigma}_A(2/0)\right)_x + m_B\,\vec{V}_A(2/0) \wedge \vec{V}_G(2/0)$$

$$= \frac{d}{dt}\left[-m_B(\ell-\ell_1)R\dot{\theta}\left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right) + I_{nn}\,\Omega(2/0)\right]\vec{x}$$

$$+ m_B\,(-R\dot{\theta})\begin{pmatrix}0\\ \cos\theta\\ \sin\theta\end{pmatrix} \wedge \begin{pmatrix}0\\ (\ell-\ell_1)\Omega(2/0)\cos\varphi - R\dot{\theta}\cos\theta\\ (\ell-\ell_1)\Omega(2/0)\sin\varphi - R\dot{\theta}\sin\theta\end{pmatrix}.$$

$$\boxed{\begin{aligned}\vec{\delta}_A(2/0) &= \frac{d}{dt}\left[-m_B(\ell-\ell_1)R\dot{\theta}\left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right) + I_{nn}\,\Omega(2/0)\right]\vec{x}\\ &+ m_B(-R\dot{\theta})\Big[\cos\theta\left[(\ell-\ell_1)\Omega(2/0)\sin\varphi - R\dot{\theta}\sin\theta\right]\\ &\qquad - \sin\theta\left[(\ell-\ell_1)\cos\varphi - R\dot{\theta}\cos\theta\right]\Big]\vec{x}\end{aligned}}$$

Il faut exprimer avec $\ddot{\theta} = 0$.

$$\frac{d}{dt}\left[-m_B(\ell-\ell_1)R\dot{\theta}\left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right) + I_{nn}\,\Omega(2/0)\right]$$

$$= \frac{d}{dt}\left[-m_B(\ell-\ell_1)R\dot{\theta}\left(\cos\theta\left(\frac{-1}{\ell}\right)\sqrt{\ell^2-R^2\sin^2\theta} + \frac{R}{\ell}\sin^2\theta\right) + I_{nn}\,\Omega(2/0)\right]$$

$$= \frac{d}{dt}\left[-m_B\frac{\ell-\ell_1}{\ell}R\dot{\theta}\left(R\sin^2\theta - \cos\theta\sqrt{\ell^2-R^2\sin^2\theta}\right) + I_{nn}\,\Omega(2/0)\right]$$

$$= -m_B\frac{\ell-\ell_1}{\ell}R\dot{\theta}\left[R\frac{d}{dt}\left(\sin^2\theta\right) - \frac{d}{dt}\left(\cos\theta\sqrt{\ell^2-R^2\sin^2\theta}\right)\right] + I_{nn}\frac{d}{dt}\left(\Omega(2/0)\right)$$

$$= -m_B\frac{\ell-\ell_1}{\ell}R\dot{\theta}\left[R\dot{\theta}\sin 2\theta + \left(\sin\theta\sqrt{\ell^2-R^2\sin^2\theta} + \frac{R^2\cos\theta\sin^2\theta}{\sqrt{\ell^2-R^2\sin^2\theta}}\right)\dot{\theta}\right]$$

$$+ I_{nn}\frac{d}{dt}\left(\Omega(2/0)\right)$$

$$\boxed{\frac{d}{dt}\left(\vec{\sigma}_A(2/0)\right) = -m_B\frac{\ell-\ell_1}{\ell}R\dot{\theta}^2\left[R\sin 2\theta + \sin\theta\left[\sqrt{\ell^2-R^2\sin^2\theta} + \frac{R^2\sin 2\theta}{2\sqrt{\ell^2-R^2\sin^2\theta}}\right]\right] + I_{nn}\frac{d}{dt}\left(\Omega(2/0)\right)}$$

inutile

et $$\frac{d}{dt}\left(\Omega(2/0)\right) = -R\dot{\theta}^2\left[\frac{-\sin\theta}{\sqrt{\ell^2-R^2\sin^2\theta}} + \frac{R^2\sin\theta\cos^2\theta}{\left(\ell^2-R^2\sin^2\theta\right)^{3/2}}\right]$$

$$\boxed{\frac{d}{dt}\left(\Omega(2/0)\right) = R\dot{\theta}^2\left[\frac{\sin\theta}{\sqrt{\ell^2-R^2\sin^2\theta}} - \frac{R^2\sin\theta\cos^2\theta}{\left(\ell^2-R^2\sin^2\theta\right)^{3/2}}\right]}$$

Détermination de l'accélération de la bielle

$$\vec{V}_G(2/0) \begin{pmatrix} 0 \\ (L-L_1)\,\Omega(2/0)\cos\varphi - R\dot\theta\cos\theta \\ (L-L_1)\,\Omega(2/0)\sin\varphi - R\dot\theta\sin\theta \end{pmatrix}$$

$$\vec{a}_G(2/0) = \frac{d}{dt}\left[\vec{V}_G(2/0)\right]$$

$$= \begin{pmatrix} 0 \\ (L-L_1)\left[\dfrac{d\Omega(2/0)}{dt}\cos\varphi + \Omega(2/0)\dfrac{d}{dt}(\cos\varphi)\right] + R\dot\theta^2\sin\theta \\ (L-L_1)\left[\dfrac{d\Omega(2/0)}{dt}\sin\varphi + \Omega(2/0)\dfrac{d}{dt}(\sin\varphi)\right] - R\dot\theta^2\cos\theta \end{pmatrix}$$

$$\boxed{\vec{a}_G(2/0) = \begin{pmatrix} 0 \\ (L-L_1)\left(\dfrac{d\Omega(2/0)}{dt}\cos\varphi - (\Omega(2/0))^2\sin\varphi\right) + R\dot\theta^2\sin\theta \\ (L-L_1)\left[\dfrac{d\Omega(2/0)}{dt}\sin\varphi + (\Omega(2/0))^2\cos\varphi\right] - R\dot\theta^2\cos\theta \end{pmatrix}_{(\vec{x},\vec{y}_0,\vec{z}_0)}}$$

Simplifions l'expression de $\dfrac{d}{dt}\left(\vec{\sigma}_A(2/0)\right)$.

$$\vec{x}\cdot\vec{\sigma}_A(2/0) = -m_B(l-l_1)R\dot\theta\left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right) + I_{xx}\,\Omega(2/0)$$

$$\frac{d\,\vec{\sigma}_A(2/0)\cdot\vec{x}}{dt} = -m_B(l-l_1)R\dot\theta\left[\frac{d\cos\theta\cos\varphi}{dt} + \frac{d\sin\theta\sin\varphi}{dt}\right] + I_{xx}\frac{d\Omega(2/0)}{dt}$$

$$= -m_B(l-l_1)R\dot\theta\left[-\dot\theta\sin\theta\cos\varphi - \Omega(2/0)\cos\theta\sin\varphi + \dot\theta\cos\theta\sin\varphi + \Omega(2/0)\sin\theta\cos\varphi\right] + I_{xx}\frac{d\Omega(2/0)}{dt}$$

$$\boxed{\frac{d\vec{\sigma}_A(2/0)\cdot\vec{x}}{dt} = -m_B(l-l_1)R\dot\theta\left[\dot\theta\left(\cos\theta\sin\varphi - \sin\theta\cos\varphi\right) + \Omega(2/0)\left[\sin\theta\cos\varphi - \cos\theta\sin\varphi\right]\right] + I_{xx}\frac{d\Omega(2/0)}{dt}}$$

Appliquons le PFS à la bielle

$$T_A(1\to 2) = \begin{pmatrix} 0 & 0 \\ Y_{12} & 0 \\ Z_{12} & 0 \end{pmatrix}_{(\vec{x},\vec{y_0},\vec{z_0})} \qquad T_B(3\to 2) = \begin{pmatrix} 0 & 0 \\ Y_{32} & 0 \\ Z_{32} & 0 \end{pmatrix}_{(\vec{x},\vec{y_0},\vec{z_0})}.$$

$$\vec{M}_A(3\to 2) = \vec{M}_B(3\to 2) + \vec{R}_D(3\to 2) \wedge \vec{BA}$$

$$= \begin{pmatrix} 0 \\ Y_{32} \\ Z_{32} \end{pmatrix}_{(x,y_0,z_0)} \wedge L \begin{pmatrix} 0 \\ -\sin\varphi \\ \cos\varphi \end{pmatrix} = L \begin{pmatrix} Y_{32}\cos\varphi + Z_{32}\sin\varphi \\ 0 \\ 0 \end{pmatrix}.$$

$$M_B\, \vec{\gamma_G}(2/0) = R_A(1\to 2) + R(3\to 2)$$

Donc
$$\boxed{\begin{cases} Y_{12} = M_B(L-L_1)\left[\dfrac{d\Omega(2/0)}{dt}\cos\varphi - (\Omega(2/0))^2\sin\varphi\right] + R\dot{\theta}^2\sin\theta - Y_{32} \\ Z_{12} = M_B(L-L_1)\left[\dfrac{d\Omega(2/0)}{dt}\sin\varphi + (\Omega(2/0))^2\cos\varphi\right] - R\dot{\theta}^2\cos\theta - Z_{32}. \end{cases}}$$

Exprimons

$$\vec{V}_B(2/0) = \vec{V}_B(3/0)$$

$$\vec{V}_B(2/0) = \vec{V}_A(2/0) + \vec{\Omega}(2/0) \wedge \vec{AB}$$

$$= -R\dot{\theta}\,\vec{y_1} - \frac{R\dot{\theta}\cos\theta}{\sqrt{l^2-R^2\sin^2\theta}}\,\vec{x} \wedge (-L)\,\vec{z_2}$$

$$= -R\dot{\theta}\,\vec{y_1} + \frac{R\dot{\theta}\cos\theta}{\sqrt{l^2-R^2\sin^2\theta}} \times L\,(-\vec{y_2}).$$

Dans (0).

$$\vec{V}_B(2/0) = -R\dot{\theta}\left(\cos\theta\,\vec{y_0} + \sin\theta\,\vec{z_0}\right) - \frac{R\dot{\theta}\cos\theta \times L}{\sqrt{l^2-R^2\sin^2\theta}}\left(\cos\varphi\,\vec{y_0} + \sin\varphi\,\vec{z_0}\right).$$

$$= -R\dot{\theta}\left(\cos\theta\,\vec{y_0} + \sin\theta\,\vec{z_0}\right) + R\dot{\theta}\cos\theta\,\vec{y_0} - \frac{R\dot{\theta}\cos\theta\, R\sin\theta}{\sqrt{l^2-R^2\sin^2\theta}}$$

$$\boxed{\vec{V}_B(2/0) = \left[-R\dot{\theta}\sin\theta - \frac{R^2\dot{\theta}\sin 2\theta}{2\sqrt{l^2-R^2\sin^2\theta}}\right]\vec{z_0}}$$

Appliquons le PFS au piston

$$m_p \frac{d\vec{V}_G(3/0)}{dt} = m_p \frac{d\vec{V}_B(2/0)}{dt} = \vec{R}(2\to3) + \vec{R}(0\to3)$$

$$m_p \frac{d\vec{V}_B(2/0)}{dt} \cdot \vec{z}_0 = Z_{23} + \underset{\substack{\| \\ 0}}{Z_{03}} \quad \text{(liaison glissière)}$$

$$\Rightarrow \quad Z_{32} = m_p \frac{d}{dt}\left[+R\dot{\theta}\sin\theta + \frac{R^2\dot{\theta}\sin 2\theta}{2\sqrt{\ell^2 - R^2\sin^2\theta}}\right].$$

$$\cancel{Z_{32} = m_p\left[-R\dot{\theta}^2\cos\theta - \frac{R^2\dot{\theta}^2}{\sqrt{L^2-R^2\sin^2\theta}}\left[\cos 2\theta - \frac{R^2}{\ldots}\right.\right.}$$

$$\boxed{Z_{32} = \left(+R\dot{\theta}^2\cos\theta + \frac{R^2\dot{\theta}^2\cos 2\theta}{\sqrt{\ell^2-R^2\sin^2\theta}} + \frac{R^4\dot{\theta}^2\sin^2 2\theta}{4\sqrt{\ell^2-R^2\sin^2\theta}^{\,3}}\right)\times m_p.}$$

Moment dynamique en A

$$\delta_A(2/0) = L\,(Y_{32}\cos\varphi + Z_{32}\sin\varphi)$$

$$\Rightarrow \boxed{Y_{32} = \frac{1}{\cos\varphi}\left[\frac{\delta_A(2/0)}{L} - Z_{32}\sin\varphi\right].}$$

Il suffit alors de réinjecter.