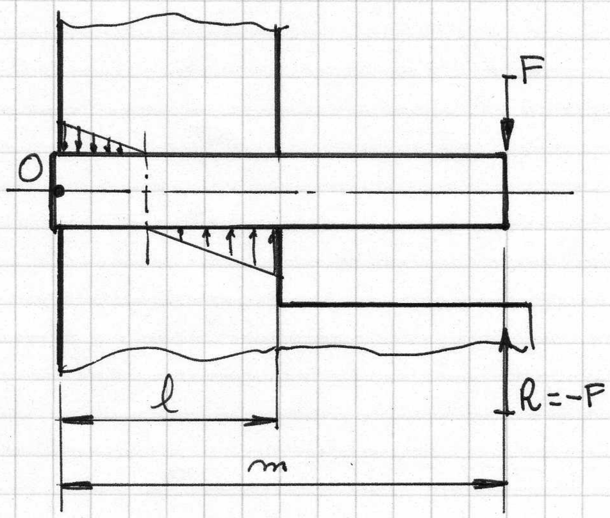
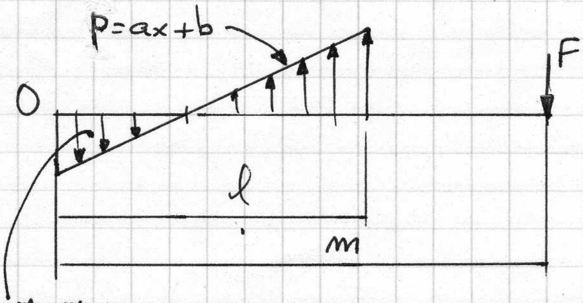
3 - Répartition longitudinale de la pression (F non centrée)

(l'axe est considéré comme rigide sur sa fibre neutre) $\rightarrow p = ax + b$



Croquis 4



$\Sigma M = M_1$
$\Sigma F = F_1$

Croquis 5

On écrit que la somme des moments$/_0$ est nulle : (croquis 5)

$$M_1 = \int_0^\ell p x . dx \quad \text{avec } p = ax + b.$$

$$M_1 = \int_0^\ell (ax+b)x . dx = \int_0^\ell (ax^2 + bx)\, dx$$

$$M_1 = \left[\frac{a}{3}x^3 + \frac{b}{2}x^2 \right]_0^\ell = \left(\frac{a\ell^3}{3} + \frac{b\ell^2}{2} \right) - (0+0)$$

$$M_1 = \frac{a\ell^3}{3} + \frac{b\ell^2}{2}$$

Comme $M_2 = -F.m.$ et $M_1 + M_2 = 0$

$$-F.m + \frac{a\ell^3}{3} + \frac{b\ell^2}{2} = 0 \qquad (1)$$

On écrit que la somme des forces est nulle :

$$F_1 = \int_0^\ell (ax+b)\, dx = \left[ax^2 + bx \right]_0^\ell$$

$$F_1 = \left(\frac{a\ell^2}{2} + b\ell \right) - (0+0) = \frac{a\ell^2}{2} + b\ell$$

et comme $F_1 + (-F) = 0$

$$-F + \frac{a\ell^2}{2} + b\ell = 0 \qquad (2)$$

$$\left.\begin{aligned} (1) \quad a &= \left(F.m - \frac{b\ell^2}{2} \right) \cdot \frac{3}{\ell^3} \\ (2) \quad a &= \left(F - b\ell \right) \cdot \frac{2}{\ell^2} \end{aligned}\right\}$$

$$\frac{3F.m}{\ell^3} - \frac{3b\ell^2}{2\ell^3} = \frac{2F}{\ell^2} - \frac{2b\ell}{\ell^2}$$

$$\frac{3b\ell^2}{2b\ell^3} - \frac{2b\ell}{\ell^2} = \frac{3F.m}{\ell^3} - \frac{2F}{\ell^2}$$

$$b\left(-\frac{2}{\ell} + \frac{3}{2\ell} \right) = \frac{F}{\ell^2}\left(\frac{3m}{\ell} - 2 \right)$$

$$b = \frac{F}{\ell^2} \left[\frac{\left(\frac{3m}{\ell} - 2 \right)}{\left(-\frac{2}{\ell} + \frac{3}{2\ell} \right)} \right] = \frac{F}{\ell^2} \left[\frac{\frac{3m}{\ell} - 2}{-\frac{1}{2\ell}} \right]$$

$$b = \frac{F}{\ell^2} \cdot 2\ell \left(\frac{3m}{\ell} - 2 \right)$$

$$\boxed{b = -\frac{2F}{\ell}\left(\frac{3m}{\ell} - 2 \right)}$$