Devoir 2013 Traitement du Signal

Exercice 1 :

Calculer la transformer de Fourier du signal h(t)

$$h(t) = (\cos(2\pi f_0 t)\, rect_T(t)) \sum_{-\infty}^{+\infty} \delta(t - nT_e)$$

$$\text{avec } T_e = 2T \text{ et } T_0 = \frac{1}{f_0} = \frac{T_e}{4}$$

$$TF[h(t)] = \mathrm{TF}[\cos(2\pi f_0 t)] * TF[rect_T(t)] * TF[\sum_{-\infty}^{\infty} \delta(t - nTe)]$$

$$TF[h(t)] = \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)] * T.sin_c(\pi f t) * \frac{1}{T_e}\sum_{-\infty}^{+\infty} \delta\,(f - \frac{n}{T_e})$$

$$TF[h(t)] = \frac{\mathrm{T}}{2}[sin_c(\pi(f - f_0)t) + sin_c(\pi(f + f_0)t)] * \frac{1}{2T}\sum_{-\infty}^{+\infty} \delta\,(f - \frac{n}{T_e})$$

$$TF[h(t)] = \frac{\mathrm{T}}{2 * 2\mathrm{T}}[sin_c(\pi(f - f_0)t) + sin_c(\pi(f + f_0)t)] * \sum_{-\infty}^{+\infty} \delta\,(f - \frac{n}{T_e})$$

$$TF[h(t)] = \frac{1}{4}[sin_c(\pi(f - f_0)t) + sin_c(\pi(f + f_0)t)] * \sum_{-\infty}^{+\infty} \delta\,(f - \frac{n}{T_e})$$

Détails des calculs :

$$\mathrm{TF}[\cos(2\pi f_0 t)] = \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

$$TF[rect_T(t)] = T sin_c(\pi f t)$$

Car :

$$rect_T(t) = u\left(t + \tfrac{T}{2}\right) - u\left(t - \tfrac{T}{2}\right) \quad et \quad rect_T{}'(t) = \delta\left(t + \tfrac{T}{2}\right) - \delta\left(t - \tfrac{T}{2}\right)$$

$$2\pi jf.TF[rect_T(t)] =$$

$$TF\left[\delta\left(t + \frac{T}{2}\right)\right] - TF\left[\delta\left(t - \frac{T}{2}\right)\right] = e^{-j2\pi f\frac{T}{2}} - e^{j2\pi f\frac{T}{2}} = e^{-j\pi fT} - e^{j\pi fT} = 2j.\sin(\pi fT)$$

Donc : $TF[rect_T(t)] = \frac{2j.\sin(\pi fT)}{2\pi jf} = \frac{\sin(\pi fT)}{\pi f} = \frac{\mathrm{T}.\sin(\pi fT)}{\pi fT} = T.sin_c(\pi f t)$

$$TF[\sum_{-\infty}^{\infty}(\delta(t - nTe)] = \frac{1}{T_e}\sum_{-\infty}^{+\infty}\delta\,(f - \frac{n}{T_e})$$