Pour trouver l'angle permettant une portée maximale on repart de :

$$x = \frac{Vi}{g} \cdot \cos\alpha \left(Vi \sin\alpha + \sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g} \right)$$

On cherche α pour x maxi, soit pour $x' = 0$

$$x = \frac{Vi}{g} \left[Vi \cdot \underset{u' = \cos\alpha}{\sin\alpha} \cdot \underset{v' = -\sin\alpha}{\cos\alpha} + \underset{u' = -\sin\alpha}{\cos\alpha} \underset{v' = \frac{1}{2\sqrt{Vi^2\sin^2\alpha + 2y_0 \cdot g}} \cdot 2\sin\alpha\cos\alpha Vi^2}{\sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g}} \right]$$

$$x' = \frac{Vi}{g} \left[Vi(\cos^2\alpha - \sin^2\alpha) + \frac{\cos\alpha \cdot \sin\alpha \cos\alpha Vi^2}{\sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g}} - \sin\alpha \cdot \sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g} \right]$$

$$x' = \frac{Vi}{g} \left[Vi(\cos^2\alpha - \sin^2\alpha) + \frac{\sin\alpha \left[Vi^2(\cos^2\alpha - \sin^2\alpha) - 2y_0 \cdot g \right]}{\sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g}} \right]$$

$$x' = \frac{Vi}{g} \left[Vi(1 - 2\sin^2\alpha) + \frac{\sin\alpha \left(Vi^2(1 - 2\sin^2\alpha) - 2y_0 \cdot g \right)}{\sqrt{Vi^2 \sin^2\alpha + 2y_0 \cdot g}} \right]$$

(en faisant $y_0 = 0$ on retrouve le résultat du cas particulier p4)

- A faire en résolution numérique pour trouver α à $x' = 0$

$\downarrow$

ou voir suite pour $\cos\alpha = f(\theta, R_0 \text{ et } L)$

Si $x' = 0$ $\quad -V_i(1-2\sin^2\alpha) = \dfrac{\sin\alpha\left(V_i^2(1-2\sin^2\alpha) - 2y_0 \cdot g\right)}{\sqrt{V_i^2\sin^2\alpha + 2y_0 \cdot g}}$

$$\sqrt{V_i^2\sin^2\alpha + 2y_0 \cdot g} = \frac{-\sin\alpha\left[V_i^2(1-2\sin^2\alpha) - 2y_0 \cdot g\right]}{V_i(1-2\sin^2\alpha)}$$

$$V_i^2\left(V_i^2\sin^2\alpha + 2y_0 \cdot g\right)(1-2\sin^2\alpha)^2 = \sin^2\alpha\left(V_i^2(1-2\sin^2\alpha) - 2y_0 \cdot g\right)^2$$

$$V_i^2\left(V_i^2\sin^2\alpha + 2y_0 \cdot g\right)(1+4\sin^4\alpha - 4\sin^2\alpha) = \sin^2\alpha\left[V_i^4(1+4\sin^4\alpha - 4\sin^2\alpha) + 4y_0^2 \cdot g^2 - 4V_i^2 \cdot y_0 \cdot g(1-2\sin^2\alpha)\right]$$

$$V_i^2\left[V_i^2\sin^2\alpha + 4V_i^2\sin^6\alpha - 4V_i^2\sin^4\alpha + 2y_0 \cdot g + 8\sin^4\alpha\, y_0 \cdot g - 8\sin^2\alpha\, y_0 \cdot g\right] = \sin^2\alpha V_i^4 + 4\sin^6\alpha V_i^4 - 4\sin^4\alpha V_i^4$$

$$+ 4\sin^2\alpha\, y_0^2 \cdot g^2 - 4\sin^2\alpha V_i^2 \cdot y_0 \cdot g + 8\sin^4\alpha . V_i^2 \cdot y_0 \cdot g$$

$$\cancel{V_i^4\sin^2\alpha} + \cancel{4V_i^4\sin^6\alpha} - \cancel{4V_i^4\sin^4\alpha} + 2V_i^2 . y_0 . g + \cancel{8V_i^2\sin^4\alpha\, y_0 . g} - 8V_i^2\sin^2\alpha . y_0 . g - \cancel{\sin^2\alpha V_i^4}$$

$$- \cancel{4\sin^6\alpha V_i^4} + \cancel{4\sin^4\alpha V_i^4} - 4\sin^2\alpha\, y_0^2 . g^2 + 4\sin^2\alpha V_i^2\, y_0 . g - \cancel{8\sin^4\alpha V_i^2\, y_0 . g} = 0$$

$$-4\sin^2\alpha V_i^2\, y_0 . g - 4\sin^2\alpha\, y_0^2 g^2 + 2V_i^2\, y_0 . g = 0$$

$$\underset{=(1-\cos^2\alpha)}{\sin^2\alpha}\left(V_i^2 + y_0 . g\right) - \frac{V_i^2}{2} = 0 \quad \text{mais } y_0 = h + L(1-\cos\alpha)$$

$$V_i^2 + y_0 g - \cos^2\alpha\left(V_i^2 + y_0 g\right) - \frac{V_i^2}{2} = 0$$

$$-\cos^2\alpha\left(V_i^2 + g(h + L(1-\cos\alpha))\right) + \frac{V_i^2}{2} + g\left(h + L(1-\cos\alpha)\right) = 0$$

$$-\cos^2\alpha\left(V_i^2 + g(h+L) - gL\cos\alpha\right) + \frac{V_i^2}{2} + g(h+L) - gL\cos\alpha = 0$$

$$\text{et } V_i^2 = 2 \cdot g \cdot L \cdot (\cos\alpha - \cos\theta)$$

$$-\cos^2\alpha\left(2\cancel{g}L\cos\alpha - 2\cancel{g}L\cos\theta + \cancel{g}\underbrace{(h+L)}_{=h_0} - \cancel{g}L\cos\alpha\right) + \cancel{\cancel{g}L\cos\alpha} - \cancel{g}L\cos\theta + \cancel{g}\underbrace{(h+L)}_{=h_0} - \cancel{\cancel{g}L\cos\alpha} = 0$$

$$-\cos^2\alpha\left(L\cos\alpha - 2L\cos\theta + h_0\right) - L\cos\theta + h_0 = 0$$

$$-\cos^2\alpha\left(\cos\alpha - 2\cos\theta + \frac{h_0}{L}\right) - \cos\theta + \frac{h_0}{L} = 0$$

$$\boxed{\cos^3\alpha - \cos^2\alpha\left(2\cos\theta - \frac{h_0}{L}\right) + \left(\cos\theta - \frac{h_0}{L}\right) = 0}$$

$$ax^3 + bx^2 + d = 0$$

oui

si $\begin{cases} a = 1 \\ b = -\left(2\cos\theta - \frac{h_0}{L}\right) \\ c = 0 \\ d = \left(\cos\theta - \frac{h_0}{L}\right) \end{cases}$

Trop compliqué $\rightarrow$ résolution numérique