Modulation and Coding - Report

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Contents

Chapter 1

Exercise 1

1.1 Question 1

For the first question, I use this Matlab script to sample all signals:

```
Fe = 8000; % Sampling frequency
Te = 1/Fe; % Sampling period
L = 8; % Number of samples
t = (0:L—1)/Fe; % Chosen times for samples
f = 0:Fe/L:Fe—Fe/L; % Chosen frenquency for samples
x = 1+0*t; % Expression of the signal
y_fft = 2*fft(x)/L; % Calculate the DFT and normalize it.


subplot(311);
stem(f,abs(y_fft)); % Plot the spectrum of the signal
xlabel('Frenquency');
ylabel('Amplitude');

subplot(312);
stem(f,real(y_fft)); % Plot the real part of the DFT
xlabel('Frenquency');
ylabel('Amplitude');

subplot(313);
stem(f,imag(y_fft)); % Plot the imaginary part of the DFT
xlabel('Frenquency');
ylabel('Amplitude');
```

For each experiment, **x** is changing. **L** is set to 8 by default, but can be redefined. In this case, it will be specified.

- **x=1, signal length 8 samples**

```
x = 1+0*t; % To make the array easily
```
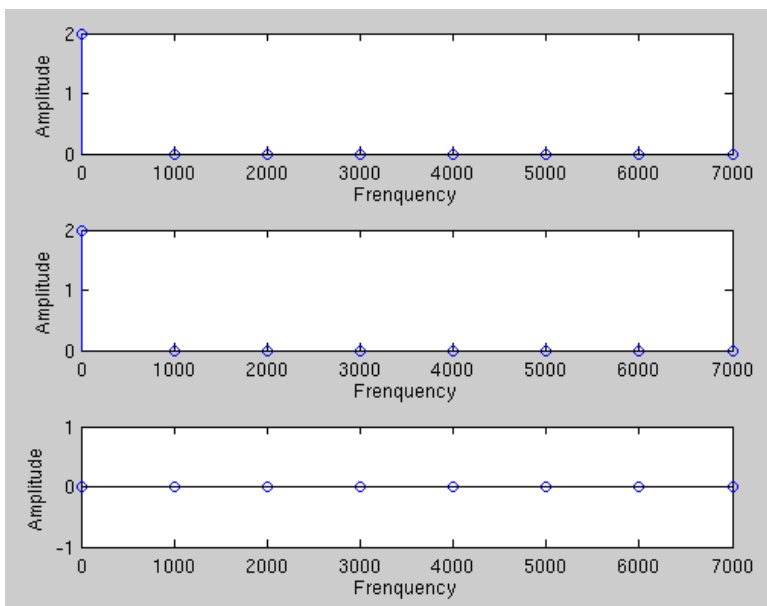

Figure 1.1: Signal's spectrum, DFT's real & imaginary parts

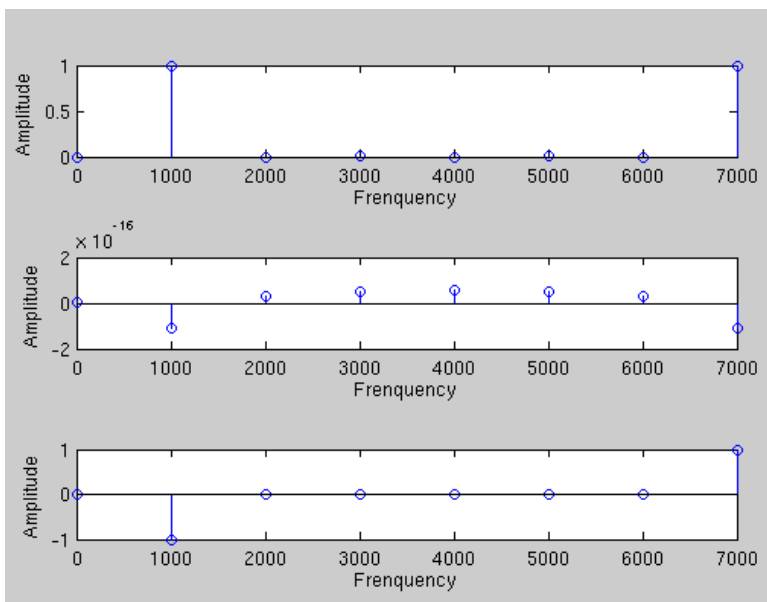- **x=sin(2π*1000*t), signal length 8 samples**

```
x = sin(2*pi*1000*t);
```


Figure 1.2: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*2000*t), signal length 8 samples**

```
x = sin(2*pi*2000*t);
```


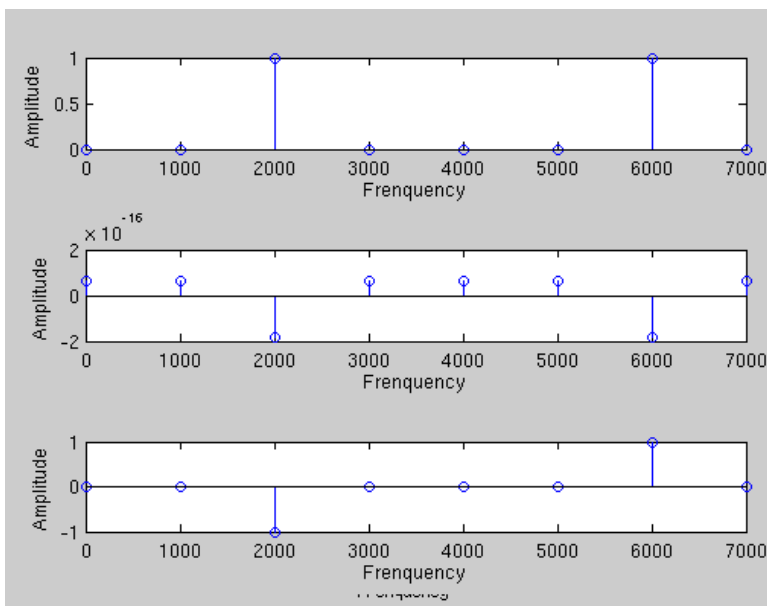Figure 1.3: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*3000*t), signal length 8 samples**

```
x = sin(2*pi*3000*t);
```
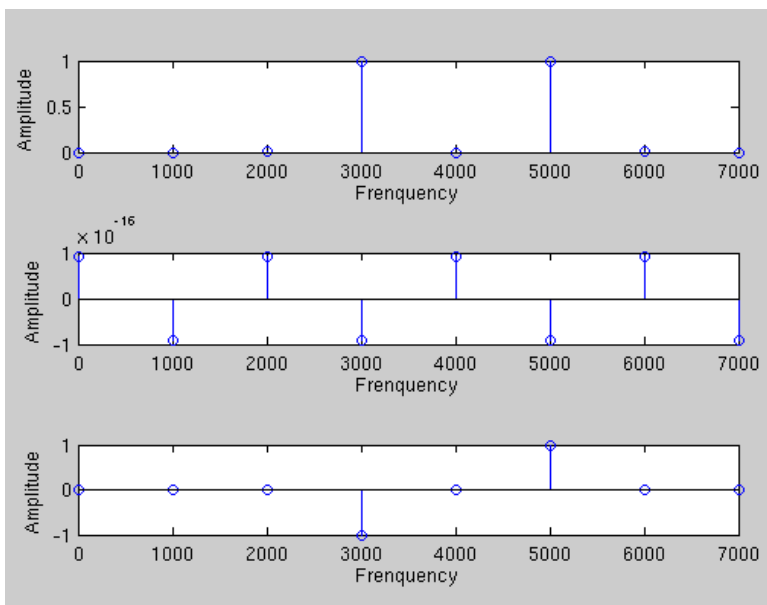

Figure 1.4: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*4000*t), signal length 8 samples**

```
x = sin(2*pi*4000*t);
```


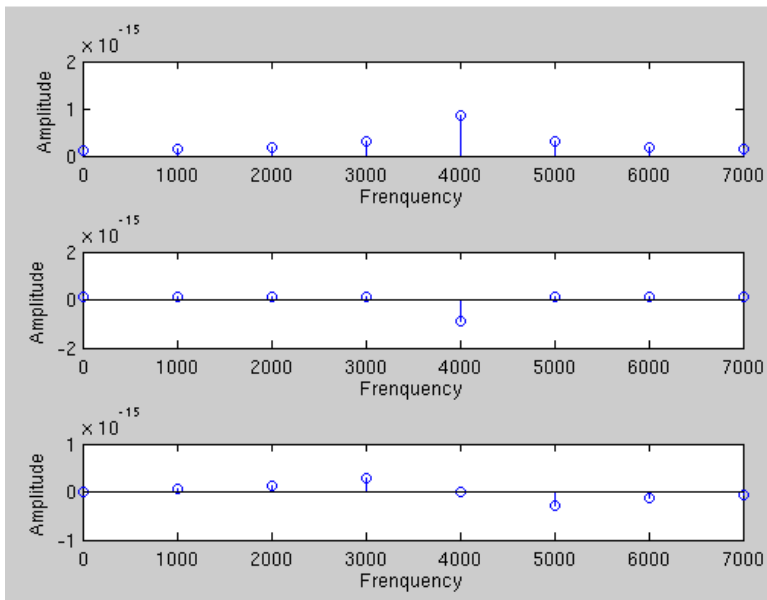
Figure 1.5: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*5000*t), signal length 8 samples**

```
x = sin(2*pi*5000*t);
```
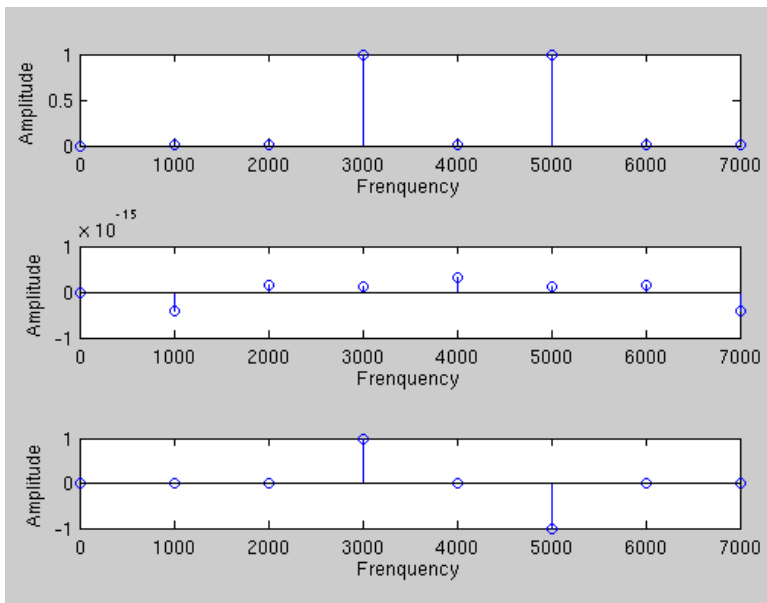

Figure 1.6: Signal's spectrum, DFT's real & imaginary parts

- **x=cos(2π*2000*t), signal length 8 samples**

```
x = cos(2*pi*2000*t);
```


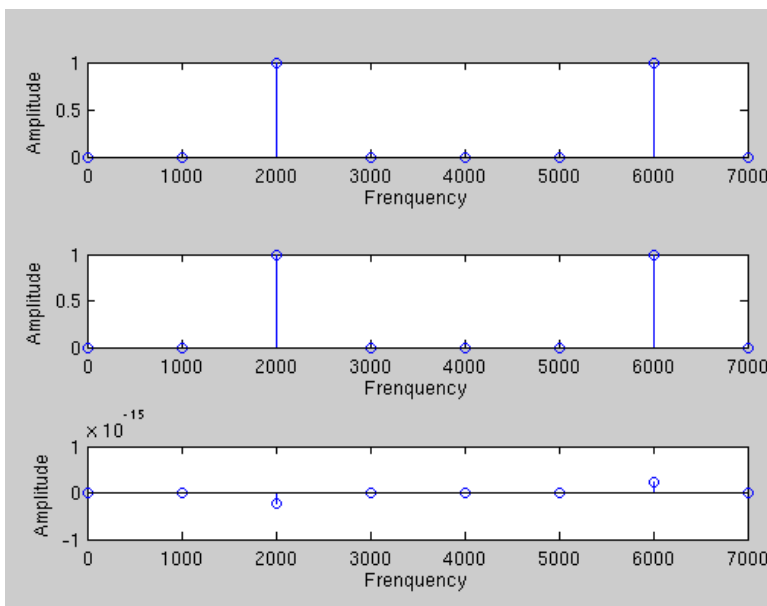
Figure 1.7: Signal's spectrum, DFT's real & imaginary parts

- **x=cos(2π*4000*t), signal length 8 samples**
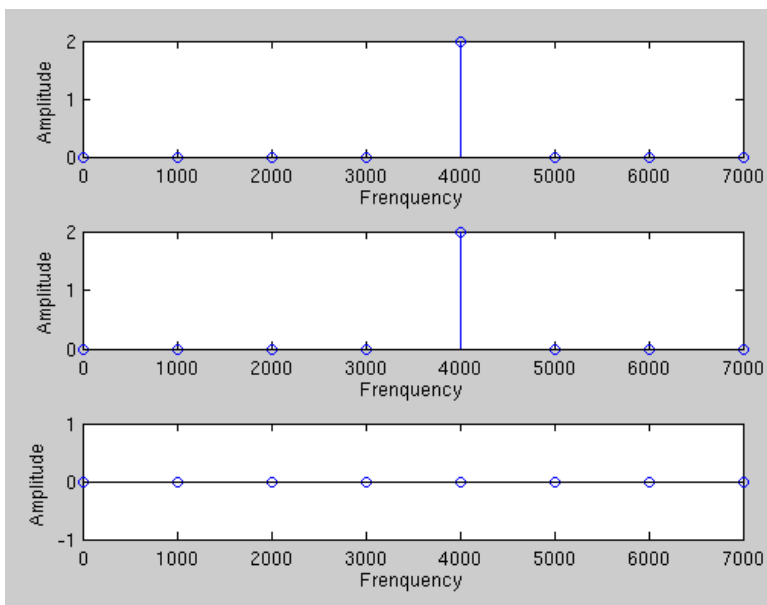
```
x = cos(2*pi*4000*t);
```


Figure 1.8: Signal's spectrum, DFT's real & imaginary parts

- **x=-1, signal length 8 samples**
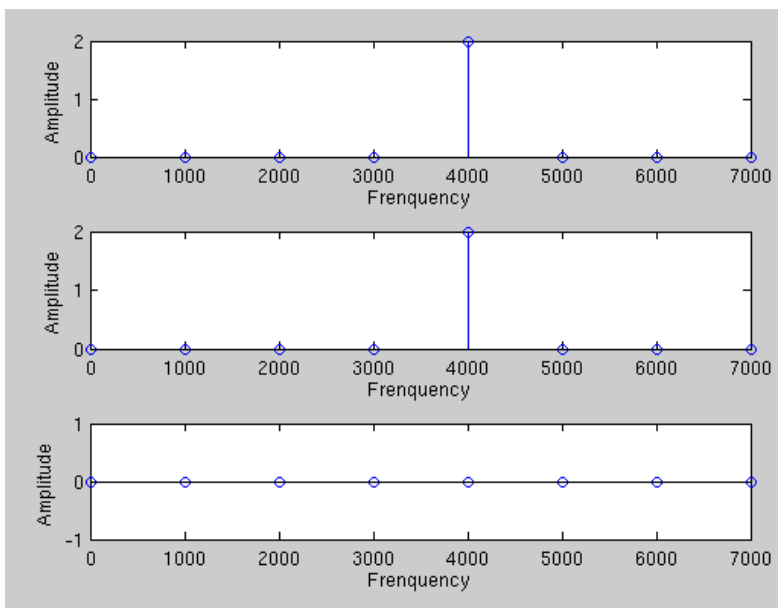
```
x = -1+0*t;
```


Figure 1.9: Signal's spectrum, DFT's real & imaginary parts

- **x=1, signal length 16 samples**

```
L = 16; % Only for this graph
x = 1+0*t;
```
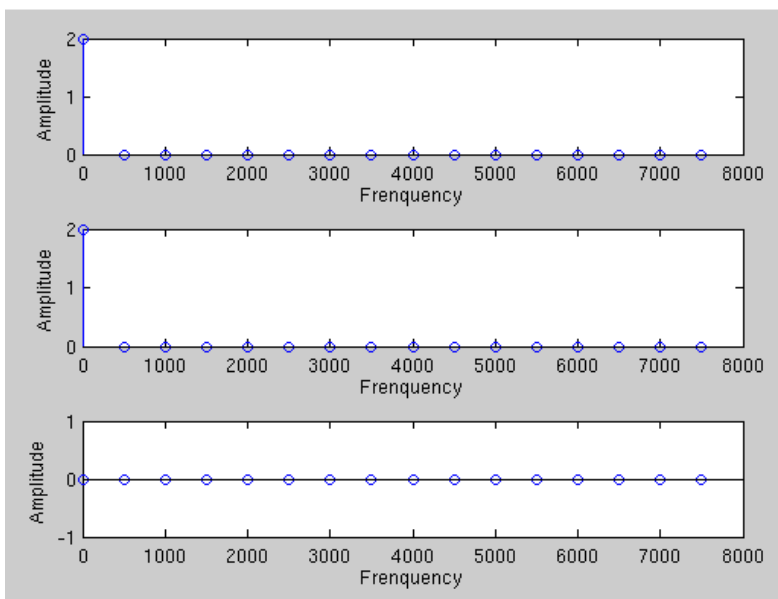

Figure 1.10: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*1000*t), signal length 16 samples**

```
L = 16; % Only for this graph
x = sin(2*pi*1000*t);
```


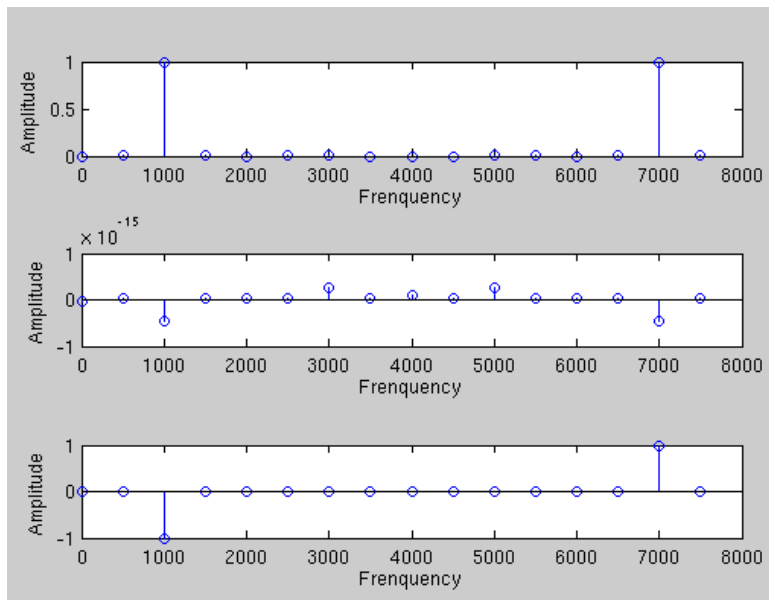
Figure 1.11: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*1000*t+0.5π), signal length 8 samples**

```
x = sin(2*pi*3000*t+0.5*pi);
```


Figure 1.12: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*1000*t), signal length 18 samples**
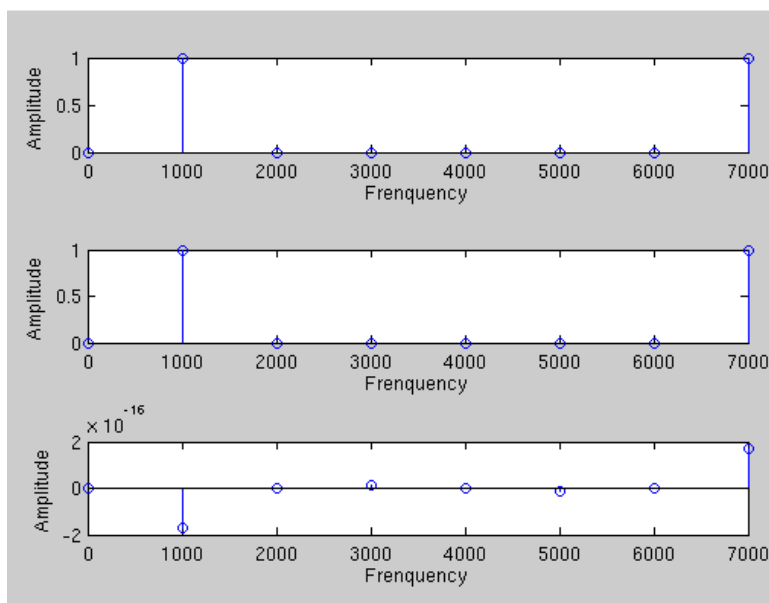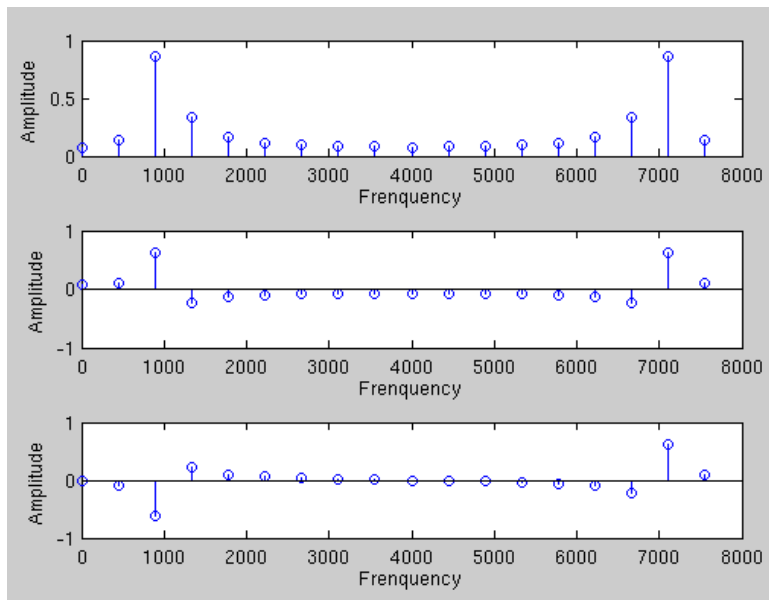
```
L = 18; % Only for this graph
x = sin(2*pi*1000*t);
```


Figure 1.13: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*1000*t), signal length 20 samples**

```
L = 20; % Only for this graph
x = sin(2*pi*1000*t);
```
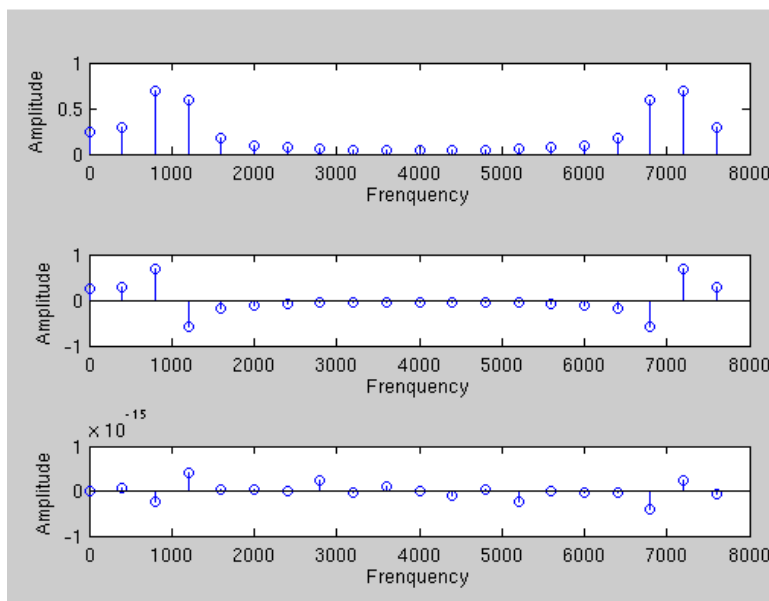

Figure 1.14: Signal's spectrum, DFT's real & imaginary parts

- **x=j*sin(2π*2000*t), signal length 8 samples**

```
x = 1i*sin(2*pi*2000*t);
```
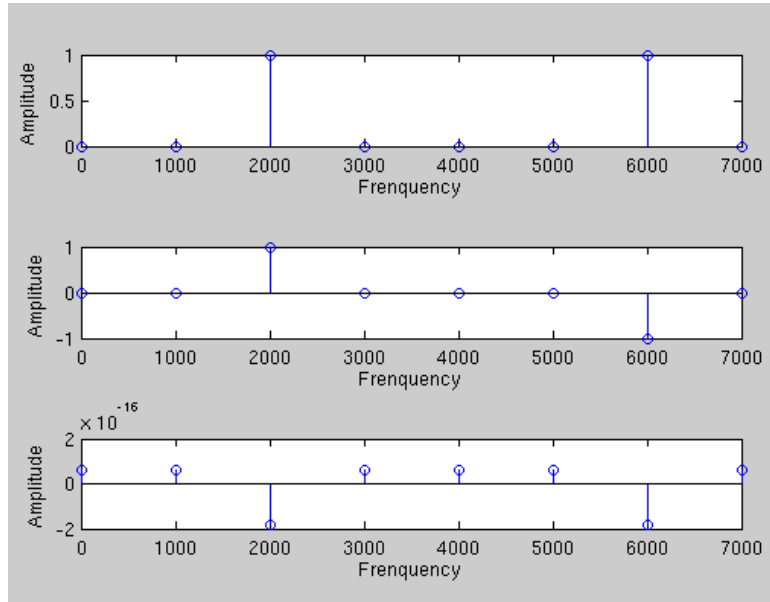

Figure 1.15: Signal's spectrum, DFT's real & imaginary parts

- **x=j*cos(2π*2000*t), signal length 8 samples**

```
x = 1i*cos(2*pi*2000*t);
```
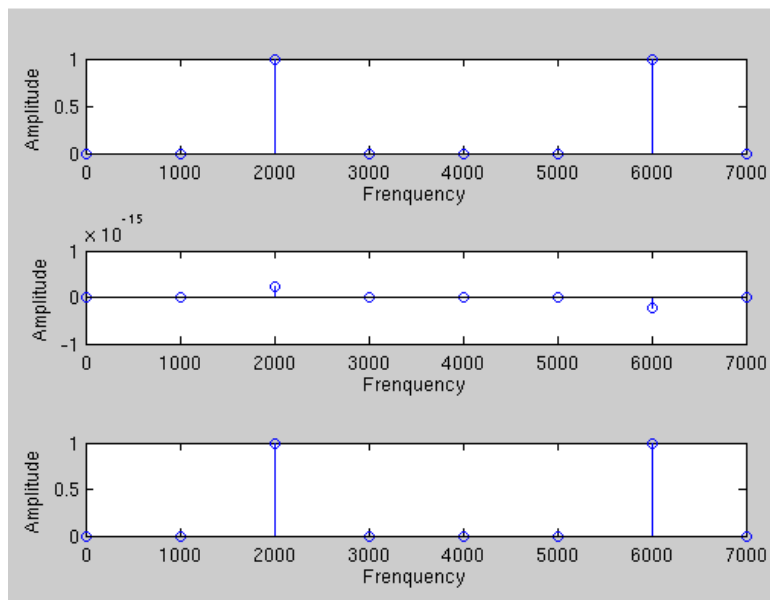

Figure 1.16: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*2000*t)+j*sin(2π*2000*t), signal length 8 samples**

```
x = sin(2*pi*2000*t)+1i*sin(2*pi*2000*t);
```
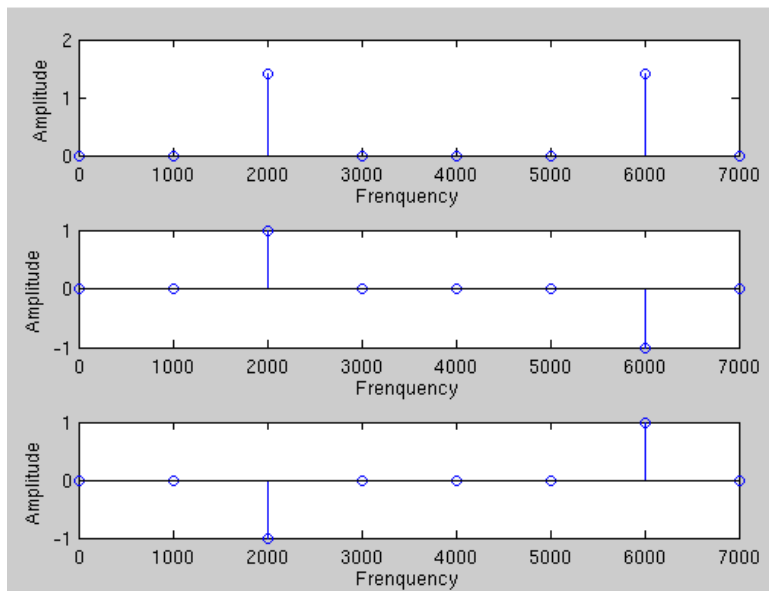

Figure 1.17: Signal's spectrum, DFT's real & imaginary parts

- **x=sin(2π*2000*t)+j*cos(2π*2000*t), signal length 8 samples**

```
x = sin(2*pi*2000*t)+1i*cos(2*pi*2000*t);
```
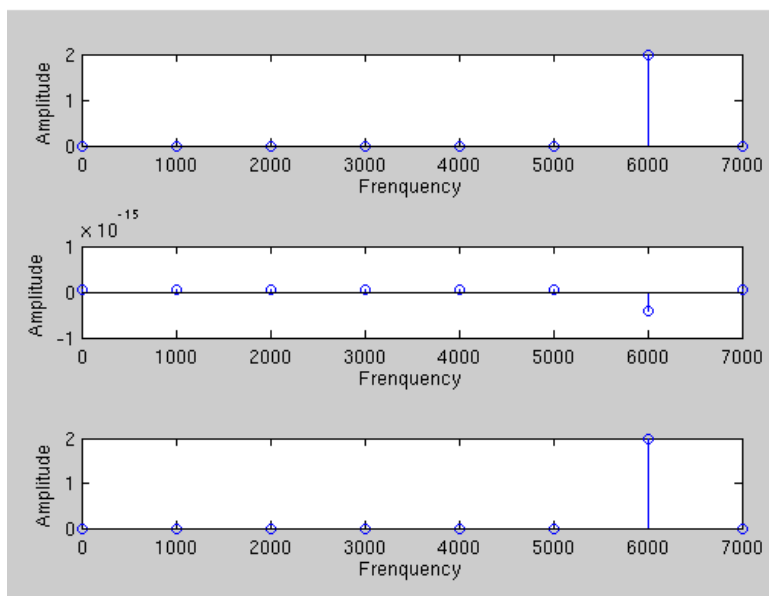

Figure 1.18: Signal's spectrum, DFT's real & imaginary parts

According to these experiments, it is possible to conclude that:

- A constant signal always has a real DFT, positive if the signal is positive, negative in the other case. There is only the first harmonic which is the double of the signal's amplitude.

- When the signal is out of phase, the spectrum doesn't change, but there is a change in the real and the imaginary part of the DFT (inversion, for a $\frac{1}{2}\pi$ change).

- If the Nyquist-Shannon (Fe > 2f) theorem is not respected for a periocal signal, there are two cases:

  - If Fe = 2f, there is a lot of harmonics, and the previous signal isn't recoverable.
  - If Fe < 2f, we find the same spectrum as some other one. (Here, the spectrum for a 1000Hz signal is the same as the spectrum for a 5000Hz signal). The unicity of the spectrum is lost.

- For a signal with a given frequency f, which respects the Nyquist-Shannon theorem, there is the first harmonic (maximum amplitude) for the frequency f, and there is an image of this harmonic for the frequency f+Fe/2.

- For a periodic signal, amplitudes in the spectrum are the amplitude of the signal for the given frequency. In the case of the presence of the image, we find only the half of the amplitude.

- If the number of samples grows, the signal is more precisely reconstructed, and the spectrum is more precise.

1.2 Question 2

For this question, I use the same script as for the first one, with some minor modifications:

```
Fe = 8000;
Te = 1/Fe;
L = 4;
t = (0:L-1)/Fe;
f = -Fe/2:Fe/L:Fe/2-Fe/L; % To center our frequency range on 0
x = % The signal we have to find
y_fft = fftshift(2*fft(x)/L); % To center the DFT on 0.

subplot(311);
stem(f,abs(y_fft));
xlabel('Frenquency');
ylabel('Amplitude');

subplot(312);
stem(f,real(y_fft));
xlabel('Frenquency');
ylabel('Amplitude');

subplot(313);
stem(f,imag(y_fft));
xlabel('Frenquency');
ylabel('Amplitude');
```

I modified the frequency range and used *fftshift* in order to center the spectrum.

According to the results of the first part, we found that, usually, the spectrum of a real periodical signal is made of frequencies which are the frequency of the signal and its image.

For a signal made of 4 samples sampled at 8kHz, the represented frequencies for a normalized (centered on 0) spectrum are -4000, -2000, 0 and 2000.

That means that we need:

- A real periodical signal with a frequency of 0Hz.

- A real periodical signal with a frequency of 2000Hz.

- A real periodical signal with a frequency of 4000Hz.

For doing that, I chose to use cosinus, and this formula: $x = cos(2\pi*0*t) + cos(2\pi*2000*t) + cos(2\pi*4000*t)$.

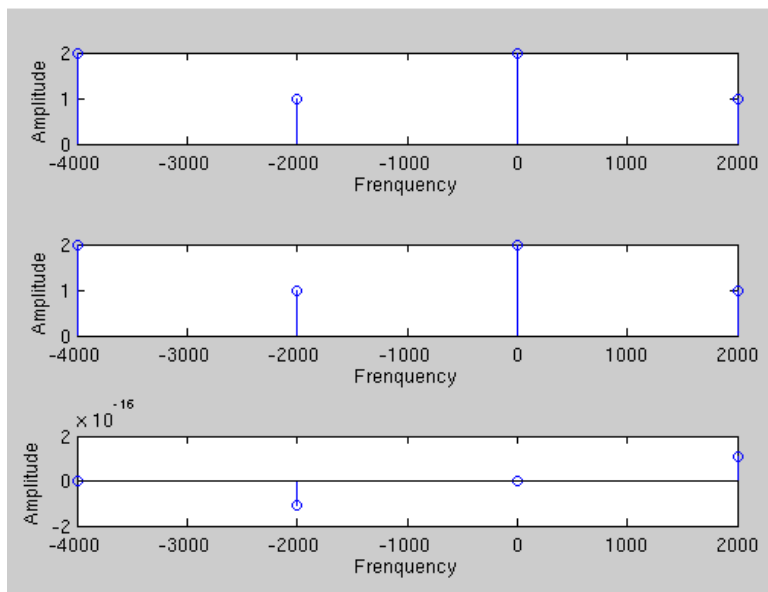In this case, the spectrum looks like this:

Figure 1.19: Signal's spectrum, DFT's real & imaginary parts

This can be explained by the fact that for the 2000Hz signal, there is 2 frequencies displayed, so the amplitude is divided by 2, according to the question 1.

The solution is to add another 2000Hz signal in the formula, to double the amplitude on this frequency. In this case, the formula is

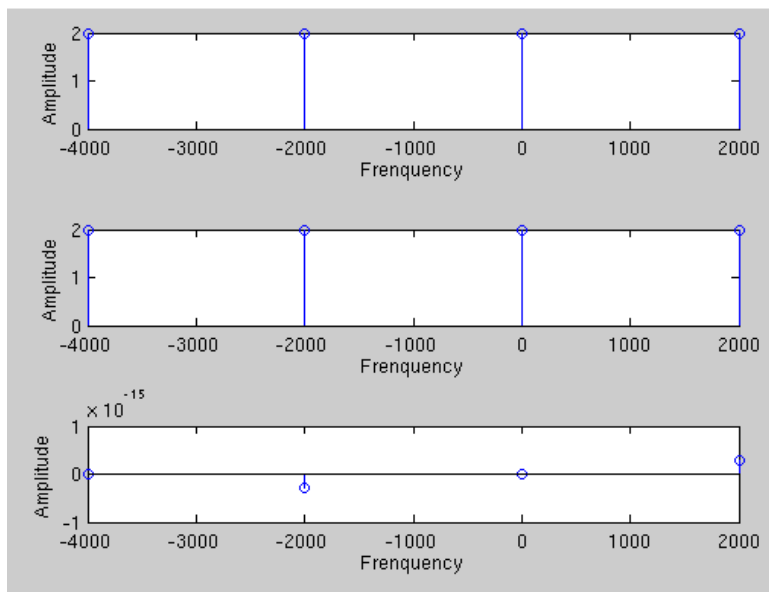$x = 1 + 2cos(2\pi*2000*t) + cos(2\pi*4000*t)$

Figure 1.20: Signal's spectrum, DFT's real & imaginary parts

There is the expected result, but with all coefficients equals to 2, because of the amplitude of our signal. To correct that, the solution is to reduce the amplitude of our signal by 2 by adding a $\frac{1}{2}$ factor on the formula.

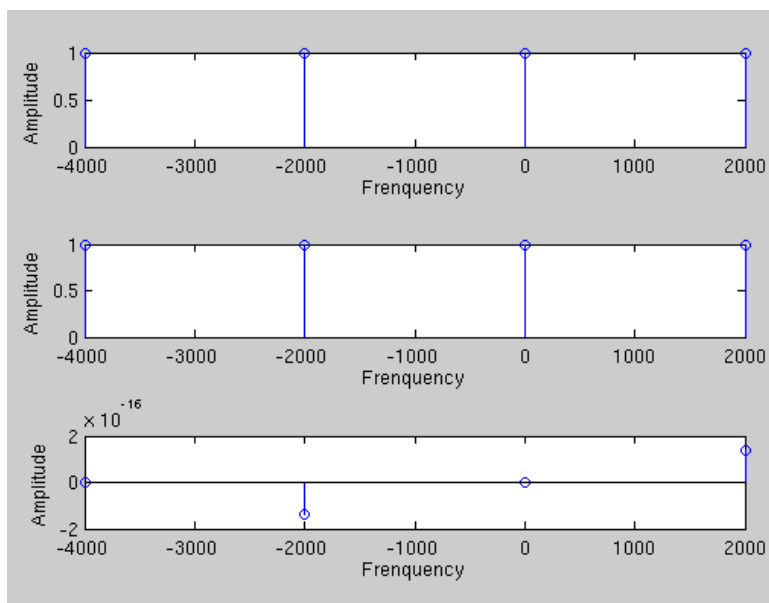$x = \frac{1}{2}(1 + 2cos(2\pi * 2000 * t) + cos(2\pi * 4000 * t))$

Figure 1.21: Signal's spectrum, DFT's real & imaginary parts

We finally get the expected spectrum for our real periodical signal.

1.3 Question 3

Not done yet.

Chapter 2

Exercise 2

First of all, the band-pass sampling theorem tells us that $\frac{Fe}{2} > B$ where Fe is the sampling frequency and B the bandwidth. The bandwidth of our signal is 12Hz so $Fe > 2 * 12 \Leftrightarrow Fe > 24Hz$.