

Snowboard jumping, Newton's second law and the force on landing

Michael J O'Shea

Cardwell Hall, Physics Department, Kansas State University, Manhattan, KS 66506-2601, USA

E-mail: mjoshea@phys.ksu.edu

Abstract

An application of Newton's second law to a snowboarder dropping off a vertical ledge shows that the average normal force during landing (force exerted by the ground on the snowboarder) is determined by four factors. It is shown that the flexing of the legs, the softness of the snow, the angle of the landing surface and the forward motion of the snowboarder can contribute significantly to reducing the force on landing. A judicious choice of the geometry of the jump leads to a force on landing that is equal to the force that the snowboarder would feel if they were standing at the landing point independent of the height from which the snowboarder jumps. Thus we are able to explain with a relatively simple model why a snowboarder may jump from rather high ledges and land comfortably. The physics here is also applicable to jumps in other sports including skiing and mountain biking. The importance of knowing the limits of models is discussed and some of the limits of this model are pointed out.

Introduction

Snowboarding began in the 1960s in the US although there were several earlier versions of people standing on a board and sliding down a snow covered slope [1, 2]. During the 1980s snowboarding rapidly gained in popularity, and it became an Olympic sport in 1998 at the Nagano winter Olympics [1]. The majority of snowboarders are teenagers. Most ski resorts now allow snowboarding and the larger ones have snowboard parks dedicated to this activity. Some of the basic snowboard techniques have similarities to skiing, such as weighting an edge to turn.

A particularly surprising part of snowboarding and skiing is the height of a ledge over which

a snowboarder or a skier can drop and still land reasonably comfortably. Analogous phenomena occur in the sports of mountain biking and dirt (motorized) biking. Such activities draw the attention of students and present a great opportunity to teach physics.

In this article we consider the forces encountered by a snowboarder on landing after a jump. Several aspects of a landing can come together to reduce the force on landing so that the person's body need only absorb a minimal impact. An important part of this modelling involves starting with the real situation and including in the model only the factors that have a significant effect on the final result. Good physical insight into the situation being studied is required to achieve this.

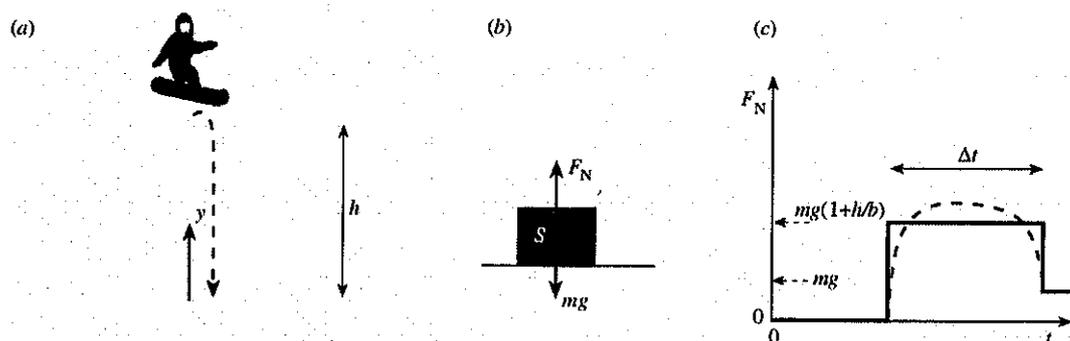


Figure 1. (a) A snowboarder with a small initial velocity drops off a ledge of height h as indicated by the dashed arrow. (b) During the landing two forces are exerted on the snowboarder (labelled S): gravity mg and the normal force of the ground on the snowboarder F_N as indicated in the free body diagram. (c) The normal force during landing is assumed constant here and the solid line shows this force as a function of time in this model. In real situations the force will not be constant but rather will rise to a maximum as shown by the dashed line so that the force we calculate here is an average value.

Body dynamics during the landing are very important and we make reasonable assumptions about how the body reacts on landing to calculate the force on landing. The form of the landing surface is also important—whether it is soft or hard and whether it is horizontal or sloping. If the snowboarder has horizontal motion the situation is somewhat more complex and we find that the horizontal motion may significantly reduce the force on landing.

Snowboarder dropping over a ledge

We consider a snowboarder jumping off a ledge of height h and landing on a hard horizontal surface, e.g. packed snow as illustrated in figure 1. The problem is to find the force exerted by the ground on the snowboarder during the landing. The important physics is in modelling how the snowboarder comes to a stop as they hit the ground. Between the times when the snowboarder first touches the ground and when they finally come to a stop, the body compresses. This compression primarily takes the form of the knees bending. This has the effect of ‘absorbing’ the force of impact so that the force is not sudden and large. While it is possible for the upper body to bend also, this effect is limited since the person must keep their centre of mass approximately over their feet to remain balanced.

The velocity v of the snowboarder when they first touch the ground can be found by looking

at the motion along the y direction or using conservation of energy [3, 4]. Assuming an initial velocity v_0 of zero, the magnitude of v is $\sqrt{2gh}$. As the snowboarder hits the ground the knees bend so that the centre of mass moves downward a distance b as they come to a stop. The y component of the velocity changes from v to a value v_f equal to zero. Assuming the acceleration, a , is constant during the landing we obtain

$$a = \frac{v_f^2 - v^2}{2(-b)} = \frac{v^2}{2b} = \frac{gh}{b}. \quad (1)$$

The acceleration is a positive quantity, indicating that it points up.

Summing forces along the y direction in figure 1(b) gives $F_N - mg = ma$, and using equation (1) we obtain the average normal force during landing:

$$F_N = mg \left(1 + \frac{h}{b} \right). \quad (2)$$

Using a height of 3 m and taking b to be approximately 0.5 m, we obtain $a = 58.8 \text{ ms}^{-2}$ and $F_N = 7mg$, a force considerably larger than that due to gravity (the weight of the person). While F_N is large it is easily absorbed by the snowboarder [5]. Note that in reality the acceleration and force during the landing will not be constant, as illustrated in figure 1(c) by the dashed line, and the maximum force felt on landing must be larger than that given by equation (2).

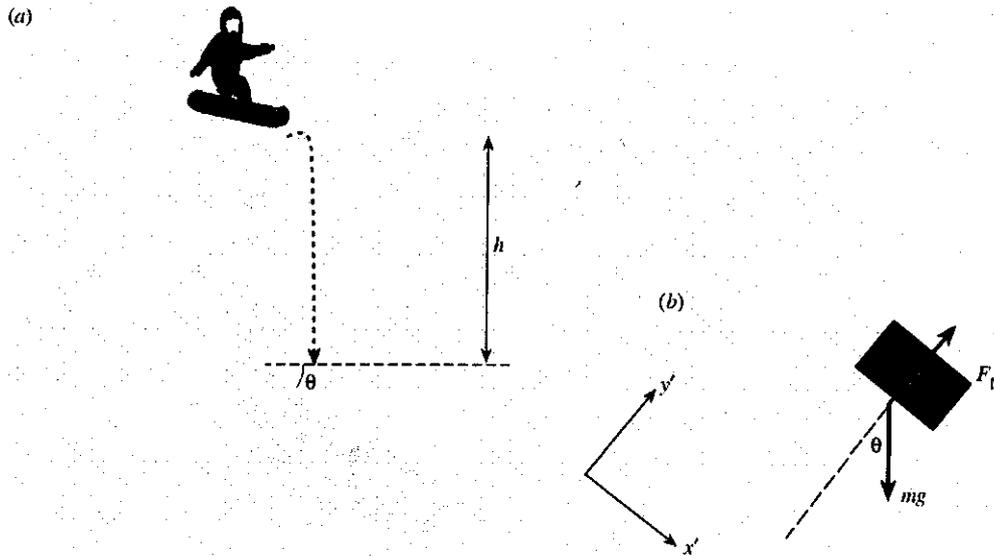


Figure 3. (a) A snowboarder with small initial velocity drops off a ledge of height h as indicated by the dashed arrow. (b) During the landing two forces are exerted on the snowboarder (labelled S): gravity mg and the normal force of the ground on the snowboarder F_N , F_N being normal to the slope as indicated in the free body diagram. It is assumed that friction with the snow is small enough to be ignored.

equation (2) is obtained. The presence of the slope reduces the normal force felt by the snowboarder on landing by a factor of $\cos \theta$. If a person jumps from a height of 3 m and lands on a 45° slope then according to equation (6) the normal force they feel on landing is $4.9mg$, a significant reduction from the force for a landing on horizontal ground.

Effect of horizontal motion

If the snowboarder has a horizontal component of velocity (towards the downhill direction) on landing then the force on landing may be significantly reduced. Recall that during landing it is the component of the velocity normal to the surface (along y') that is reduced to zero. If the snowboarder's velocity is at some angle ϕ to the horizontal on landing (see figure 4) then this component is

$$v'_y = -v \sin(\phi - \theta). \tag{7}$$

If the snowboarder descends vertically, $\phi = 90^\circ$ and $v'_y = -v \cos \theta$. While the velocity on landing may be increased for the snowboarder with some horizontal velocity the slope θ can be selected so that the sine term is close to zero, making v'_y close to zero. Thus it is possible for the

snowboarder with some horizontal velocity to have a very small change in velocity on landing and therefore experience a small normal force.

So now the question is—can this be used in real situations to reduce the force on landing? To look at this consider a vertical drop with a horizontal section before the downward slope as shown in figure 4. The snowboarder goes over the edge of the drop with a velocity $v_0 = v_{0x}$. This velocity is chosen so that the snowboarder lands at the lip of the slope as shown by the dashed curve and arrow and so v_{0x} is determined by h and d .

The time t to reach the landing point may be calculated from the motion along the vertical direction and is $\sqrt{2h/g}$, assuming an initial velocity along y of zero. The x and y components of the velocity at the landing point can now be obtained as $v_x = v_{0x} = d/t = d\sqrt{g/2h}$ and $v_y = v_{0y} - gt = -\sqrt{2gh}$ as before. Using these results the magnitude v and the direction ϕ of the velocity as the snowboarder touches the ground can be found:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{g \left(\frac{d^2}{2h} + 2h \right)} \tag{8}$$

$$\tan \phi = \frac{-v_y}{v_x} = \frac{2h}{d}.$$



Figure 2. Heading straight for the downslope. (Photograph courtesy George Manning, Saskatoon, Canada.)

The time over which this force is exerted can be calculated from the y component of the average velocity during landing $v_{av} = (v + v_f)/2 = v/2$. Using $y \equiv -b = v_{av} \Delta t$ we obtain¹:

$$\Delta t = \frac{-2b}{v} = b \sqrt{\frac{2}{gh}}. \quad (3)$$

For the above case we obtain $t = 0.130$ seconds.

If the snowboarder lands in soft snow then the force the ground exerts is 'cushioned'. This can be incorporated into the calculation by including it in the distance b . If the depth the snowboarder sinks into the snow is 0.1 m then b is 0.6 m, a is 49 m s^{-2} and $F_N = 6mg$, so that the average force on the snowboarder during landing is reduced. The advice here is just what your intuition would tell you: landing on soft snow helps!

Landing on a sloping surface

We consider a snowboarder dropping off a ledge onto a slope that is inclined at an angle θ to the horizontal as shown in figure 3. The snowboarder does not come to rest after landing on the slope because the component of the velocity parallel to the slope is not reduced to zero. Only the component of the velocity perpendicular to the

¹ A check on these results can be performed by writing Newton's second law in the form $F = \Delta P / \Delta t$. Applying this to the landing, remembering that F is the total force $F_N - mg$, and using equations (2) and (3), $F \Delta t$ is $m\sqrt{2gh}$. The change in momentum ΔP (impulse) during landing gives the same result $m(v_f - v) = m\sqrt{2gh}$.

slope is reduced to zero on landing. Since this is a two-dimensional problem we distinguish between the x and y components of vectors with a subscript. Once more the speed when the snowboarder first touches the ground is $v_y = -\sqrt{2gh}$. We now switch to the rotated x' - y' coordinate system shown in figure 3 for convenience. The component of the velocity normal to the slope is along y' and is given by $v'_y = v_y \cos \theta = -\sqrt{2gh} \cos \theta$.

We assume once more that during the landing the snowboarder's centre of mass drops a vertical distance b as the snowboarder's knees bend and that the final velocity v'_{fy} along y' is zero. We also assume the snow is hard packed and smooth so that there is no significant frictional force. The acceleration a' along y' is

$$a' = \frac{v'^2_{fy} - v'^2_{iy}}{2(-b) \cos \theta} = \frac{v_y^2}{2b \cos \theta}. \quad (4)$$

Application of Newton's second law along y' gives $F_N - mg \cos \theta = ma'$, which yields

$$F_N = m(a' + g \cos \theta) = m \left(\frac{v_y^2}{2b \cos \theta} + g \cos \theta \right). \quad (5)$$

Substituting for v'_y gives

$$F_N = mg \left(1 + \frac{h}{b} \right) \cos \theta \quad (6)$$

for the value of the normal force on landing. Notice that if $\theta = 0$ (landing on a flat surface)

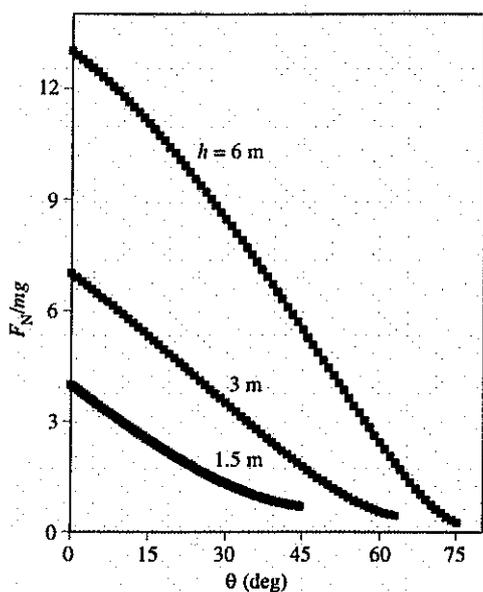


Figure 5. The force on landing as a function of slope angle θ for selected ledge heights h for the geometry of figure 4. The quantities d and b are taken to be 3 m and 0.5 m respectively, and the snowboarder launches with the correct velocity v_{0x} to land at the top of the slope.

v'_y decreases, making the change in velocity on landing smaller. The plots of F_N extend up to a maximum value of θ given by $\phi = \theta$. Values of several calculated parameters are given in table 1. The smallest value of F_N occurs at the maximum value of θ and corresponds to the special case (1) above. Notice that the minimum possible value of the force on landing actually decreases with increasing height! Of course the total speed on landing, v , increases so that a higher degree of skill is required to control the landing.

Table 1. Calculated values of the initial horizontal velocity v_{0x} , the velocity v and its angle ϕ with the horizontal on landing, the normal force F_N for landing on horizontal ground, and the minimum possible value of F_N obtained when $\theta = \phi$. Calculations are for the geometry of figure 6 for selected values of h given in the figure and for $d = 3$ m.

h (m)	v_{0x} (m s ⁻¹)	v (m s ⁻¹)	ϕ (deg)	F_N/mg	
				$\theta = 0$	$\theta = \phi$
1.5	5.42	7.67	45.0	4.0	0.71
3.0	3.84	8.58	63.4	7.0	0.45
6.0	2.71	11.18	76.0	13.0	0.24

Discussion

We have looked at the average force on landing for a snowboarder who drops over a ledge. The flexing of the legs on landing, the firmness of the snow, the slope of the landing surface and the forward speed of the snowboarder are all important in reducing the force on landing. This force has its minimum value, $mg \cos \theta$, when $\phi = \theta$ for the model jump of figure 4. This is the same normal force a person would feel if they were standing stationary at the landing point.

We have ignored possible effects of drag here. Drag may be estimated from $F_d = \frac{1}{2}k\rho Av^2$. Here k is a constant which is approximately 1, ρ is the density of air and A is the area the falling object presents to the air [6]. For the largest speeds in our examples, drag forces are less than 3% of the gravitational force and to a first approximation may be neglected. Aerodynamic forces can exist when an object with a curved surface moves through the air [7] but for the speeds of this work they are also not important. Such drag and lift forces do become substantial at the higher speeds attained for higher jumps and may need to be included.

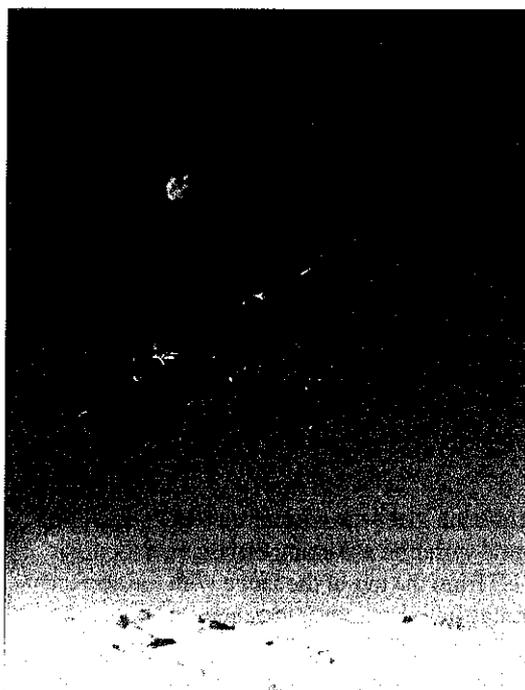


Figure 6. Looking for a landing spot. (Photograph courtesy George Manning, Saskatoon, Canada.)

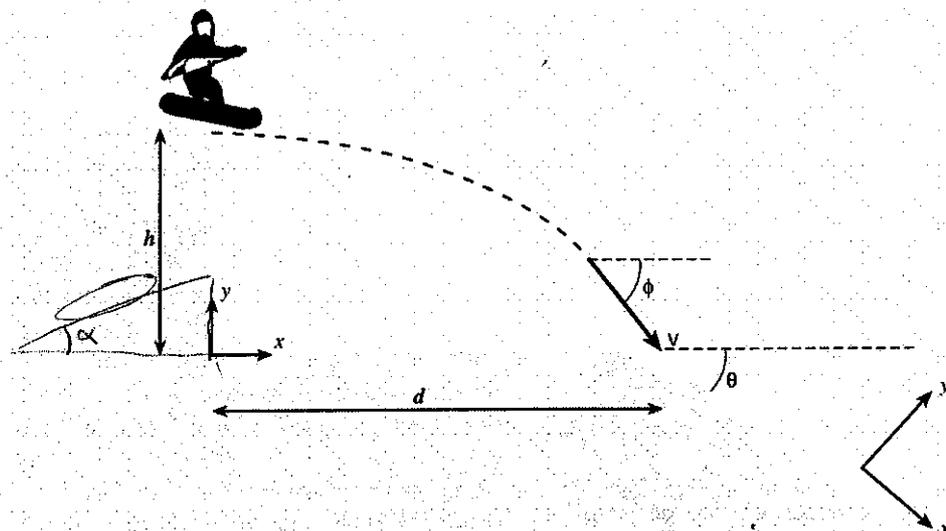


Figure 4. A snowboarder with an initial horizontal velocity jumping from a ledge and landing at the top of a slope with a velocity vector v at some angle ϕ to the horizontal. The initial velocity v_{0x} is chosen so that the snowboarder lands at the top of the slope. The free body diagram of figure 3 applies to this situation.

Note that ϕ is measured below the horizontal, hence $-v_y$ appears in the second equation above. In order for it to be possible to land at the lip of the slope, the condition $\phi \geq \theta$ must hold. Using equation (8) this implies that the condition $h > (d/2) \tan \theta$ must hold. If $\phi < \theta$ then the snowboarder will skim over the lip of the slope and land further down the slope and the following results do not apply. Using equations (5), (7) and (8) we obtain for the normal force

$$F_N = mg \cos \theta \times \left[\left(\frac{d^2}{2h} + 2h \right) \frac{\sin^2[\arctan(2h/d) - \theta]}{2b \cos^2 \theta} + 1 \right]. \quad (9)$$

This equation gives the normal force as a function of several variables assuming that the snowboarder takes off from the top of the ledge with the correct horizontal initial velocity to reach the lip of the slope.

It is instructive to look at some special cases of this general result, equation (9).

- (1) $\theta = \phi$ [$\theta = \arctan(2h/d)$]. This is the case where the velocity vector v of the snowboarder is parallel to the slope and equation (9) gives F_N as $mg \cos \theta$. The snowboarder experiences the same force on landing as they

would if they were standing stationary at the lip of the slope, a gentle landing indeed! Note that this result can be obtained directly from equation (5) since the velocity on landing is parallel to the slope, and therefore v'_y is zero.

(2) $\theta = 45^\circ$. For this case equation (9) may be simplified² to give

$$F_N = \frac{mg}{\sqrt{2}} \left(\frac{(2h-d)^2}{4bh} + 1 \right). \quad (10)$$

This force is smallest when $2h = d$, which corresponds to $\phi = \theta$.

To give further insight into the dependence of F_N on the geometry we have plotted in figure 5 the force during landing as a function of the slope θ for the model jump of figure 4. Results are given for three selected ledge heights h : 1.5 m, 3 m and 6 m. The length d is held fixed at 3 m. F_N decreases with increasing θ since, according to equation (7),

² We have used $\sin(A-B) = \sin A \cos B - \cos A \sin B$, $\cos(\arctan k) = 1/\sqrt{1+k^2}$ and $\sin(\arctan k) = k/\sqrt{1+k^2}$. These latter arctan expressions can be calculated straightforwardly. Start with the identity $\tan(\arctan k) = k$. Then using the definition of the tan function, $\sin(\arctan k)/\cos(\arctan k) = k$. Rearranging and using $\cos^2(\arctan k) = 1 - \sin^2(\arctan k)$ gives $\sin^2(\arctan k) = k^2(1 - \sin^2(\arctan k))$. Rearranging this yields the relation for $\sin(\arctan k)$. The relation for $\cos(\arctan k)$ may be obtained in a similar way.

It is possible to modify the shape of the slope to reduce the force on landing to approximately zero. This can be done with a parabolic shape as opposed to a constant slope. Such a slope would have the shape of a projectile trajectory under gravitational influence. Provided the speed is chosen correctly, the snowboarder would follow a trajectory close to the shape of the slope and could land very gently. Of course the landing surface cannot continue to follow this parabolic shape, it must eventually level off. When it does level off there will be an increase in the normal force on the snowboarder.

Skiers are also able to descend large drops relatively safely provided they have attained the high skill level required to execute the jump. The above results apply to these situations. While telemark skiers may flex their legs in a similar way to snowboarders, downhill skiers are not able to flex their legs as much due to the rigid boots they wear. In this latter case a landing on a steep slope after a big jump is important to reduce the force. In the case of mountain bikes the cushioning of the inflated tyres and the shock absorbers reduce the force on the rider on landing and can be taken account of in the parameter b , assuming the shock absorbers do not bottom out. It must be noted in all these cases that the calculation yields the average force on landing and that the maximum value of

this force will be somewhat larger than this average value.

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Michael J O'Shea is a professor at Kansas State University, where he has been since 1983, having obtained his PhD in 1981 at Sussex University. His research involves preparation and magnetic properties of nanostructured rare-earth-based permanent magnets.