

These equations can be applied quite well under atmospheric conditions, if, instead of $4y_G$ in Equation (16), one uses the quantity $3y_G$ (DUFRENOIS) [2]: Equations (16) and (17) then become the Equations

$$\Delta X = \Delta y \cot \omega \left(1 + \frac{\Delta y}{3y_G} \right) \quad (18)$$

and

$$\Delta X = \Delta y \cot \omega \left(1 + \frac{4 \Delta y \cot \omega}{3X} \right) \quad (19)$$

or, based on the investigations of W. KRAUS [3],

$$\Delta X = \Delta y \cot \omega \left(1 + \frac{3 \Delta y \cot \omega}{2X} \right). \quad (20)$$

3.2 The Trajectory in the Atmosphere

3.2.1 Aerodynamics of the Projectile

In addition to gravity, in the atmosphere additional air forces act on the projectile, which unlike gravity are surface forces. The projectile is surrounded by the atmosphere, which flows along the surface of the projectile when in flight. Thus, a pressure force is exerted perpendicular to each surface element; due to the adhesion of air molecules to the surface, frictional forces arise in a tangential direction, as a consequence of the internal friction of the air.

3.2.1.1 Air Resistance (Drag)

The Prandtl expression for an air resistance law reads

$$W = c_w \frac{\rho}{2} v^2 \frac{\pi}{4} d^2, \quad (21)$$

where

- W is the air resistance (the drag),
- c_w is the dimensionless drag coefficient,
- ρ is the air density,
- v is the projectile velocity,
- d is the diameter of the projectile.

One can easily see from the basic structure of this law that the drag W is proportional to the mass of air which the projectile must displace per unit of time, thus,

$$W \sim \rho v F, \quad (22)$$

where F is the cross-sectional area of the projectile.

Since in accordance with Newton's law of motion, the force is equal to the rate of change of momentum, the resistance is proportional overall to the product of "air mass times velocity". With the proportionality factor c_w , one then obtains Equation (21).

The dimensionless drag coefficient c_w is in no way constant, but depends on the shape and velocity of the projectile, and the state of the air through which the projectile must fly; it thus changes along the trajectory.

From analyses using dimensionless quantities, according to which Equation (21) can also be derived, one finds that the drag coefficient c_w again depends on other dimensionless quantities, the Mach number

$$M = \frac{v}{a} \quad (23)$$

and the Reynolds number

$$Re = \frac{v l_x \rho}{\eta} \quad (24)$$

(Section 3.2.1.3), where

l_x is a characteristic length dependent on the projectile's geometry (for example, the diameter d or the length l),

a is the sound velocity,

ρ is the air density,

η is the dynamic viscosity of the air.

The Mach number is a measure of how strong the inertial forces are in comparison with forces which arise from compression of the air through which the projectile flies. The Reynolds number indicates the ratio of the inertial to the frictional forces.

If the tangent to the trajectory of a projectile coincides with its axis, only one air force arises in the case of rotationally symmetrical bodies, which is comprised of the pressure or wave resistance, the frictional resistance and the wake behind the base.

