These equations can be applied quite well under atmospheric conditions, if, instead of $4y_G$ in Equation (16), one uses the quantity $3y_G$ (DUFRÉNOIS) [2]; Equations (16) and (17) then become the Equations

$$\Delta X = \Delta y \cot\omega \left(1 + \frac{\Delta y}{3y_G}\right) \tag{18}$$

and

$$\Delta X = \Delta y \cot\omega \left(1 + \frac{4\Delta y \cot\omega}{3X}\right) \tag{19}$$

or, based on the investigations of W. KRAUS [3],

$$\Delta X = \Delta y \cot\omega \left(1 + \frac{3\Delta y \cot\omega}{2X}\right). \tag{20}$$

3.2 The Trajectory in the Atmosphere

3.2.1 Aerodynamics of the Projectile

In addition to gravity, in the atmosphere additional air forces act on the projectile, which unlike gravity are surface forces. The projectile is surrounded by the atmosphere, which flows along the surface of the projectile when in flight. Thus, a pressure force is exerted perpendicular to each surface element; due to the adhesion of air molecules to the surface, frictional forces arise in a tangential direction, as a consequence of the internal friction of the air.

3.2.1.1 Air Resistance (Drag)

The Prandtl expression for an air resistance law reads

$$W = c_w \frac{\varrho}{2} v^2 \frac{\pi}{4} d^2, \tag{21}$$

where
- W is the air resistance (the drag),
- c_w is the dimensionless drag coefficient,
- ϱ is the air density,
- v is the projectile velocity,
- d is the diameter of the projectile.

One can easily see from the basic structure of this law that the drag W is proportional to the mass of air which the projectile must displace per unit of time, thus,

$$W \sim \varrho v F, \tag{22}$$

where F is the cross-sectional area of the projectile.

Since in accordance with Newton's law of motion, the force is equal to the rate of change of momentum, the resistance is proportional overall to the product of "air mass times velocity". With the proportionality factor c_w, one then obtains Equation (21).

The dimensionless drag coefficient c_w is in no way constant, but depends on the shape and velocity of the projectile, and the state of the air through which the projectile must fly; it thus changes along the trajectory.

From analyses using dimensionless quantities, according to which Equation (21) can also be derived, one finds that the drag coefficient c_w again depends on other dimensionless quantities, the Mach number

$$M = \frac{v}{a} \tag{23}$$

and the Reynolds number

$$\mathrm{Re} = \frac{v l_x \varrho}{\eta} \tag{24}$$

(Section 3.2.1.3), where
- l_x is a characteristic length dependent on the projectile's geometry (for example, the diameter d or the length l),
- a is the sound velocity,
- ϱ is the air density,
- η is the dynamic viscosity of the air.

The Mach number is a measure of how strong the inertial forces are in comparison with forces which arise from compression of the air through which the projectile flies. The Reynolds number indicates the ratio of the inertial to the frictional forces.

If the tangent to the trajectory of a projectile coincides with its axis, only *one* air force arises in the case of rotationally symmetrical bodies, which is comprised of the pressure or wave resistance, the frictional resistance and the wake behind the base.

According to this, the drag coefficient is theoretically broken down as

$$c_w = c_{wW} + c_{wR} + c_{wB} \tag{25}$$

where

c_{wW} is the contribution of the wave resistance to the c_w,
c_{wR} is the contribution of the frictional resistance to the c_w,
c_{wB} is the contribution of the wake behind the base to the c_w.

Estimates of these factors are found in I. SZABO [4], among others.

For subsonic speeds ($M < 1$), the waves run ahead of the projectile; there thus exists a certain pressure equalization. For supersonic speeds ($M > 1$), this is not possible; shock fronts are formed. Thus, the pressure or shock wave resistance in the supersonic range makes a substantial contribution to the overall drag. For subsonic velocities, friction and the wake behind the base are the primary components of the drag.

Within a very thin layer, the Prandtl boundary layer, the air molecules are carried along; they acquire a greater velocity than those molecules which are farther away from the projectile. Only at small Reynolds numbers Re are the flow lines nearly parallel to the wall (laminar boundary layer), and at large Reynolds numbers Re, vortices are formed within the boundary layer due to the internal friction of the air (turbulence), which again break up at the base of the projectile, and form a vortex path behind the projectile base (Kármán vortex path). They are, to a large extent, responsible for the size of the wake behind the base. At subsonic speeds, vortex formation can largely be suppressed by suitable shaping of the projectile (dirigible shape). The practical realization of the dirigible shape, however, is subject to certain difficulties [5].

It proves to be the case that for thin pointed projectiles, the base wake component can be up to half of the total drag, well up into the supersonic range, while with increasingly blunter projectiles, the shock wave resistance outweighs the frictional resistance.

In the normal case, where c_w corresponds approximately to the resistance coefficient c_{w1}, based on the Rheinmetall resistance law and/or the German resistance law Nr. 2 (Figure 306), at a

Mach number of 3 the breakdown of the individual coefficients is as follows:

$$\begin{aligned} c_{wW} : c_{wR} : c_{wB} &= 0.13 : 0.05 : 0.11 \\ &= 45\% : 17\% : 38\% . \end{aligned}$$

Figure 306 shows the c_w values for various projectile shapes as a function of the Mach number M.

In solving the main equation in External Ballistics (Section 3.2.2) special laws of resistance (SIACCI, d'ANTONIO) are used as well. For instance, in this form, the equation

$$W = c_n \frac{\varrho}{2} v_0^{2-n} v^n \frac{\pi}{4} d^2 \tag{26}$$

makes closed integration of the main equation possible (in d'ANTONIO $n = 1.5$). The coefficients c_n remain dimensionless because of the factor v_0^{2-n}.

Numbered among these formulas of the laws of resistance is also the relationship

$$W = m c f(v) , \tag{27}$$

where the drag W is determined as a function of the projectile velocity v, by means of the function $f(v)$. The quantity c is called the ballistic coefficient.

3.2.1.2 Aerodynamic Forces in the Case of Non-axial Flow

In contrast to the hypotheses set up in Section 3.2.1.1, in reality, the axis of the projectile always makes an angle to the direction of flight (angle of incidence α, Figure 307). In oblique incident flow the component of force acting perpendicular to the drag W is called lift A. The resolving of the aerodynamic force in the direction of flow (flight direction) and perpendicular to it, i.e. into drag and lift, can also be replaced by resolving in the direction of the projectile axis, and perpendicular to it. In this way, one obtains the tangential force T and the normal force N. Since the center of pressure D does not normally coincide with the center of gravity S, one obtains also a turning moment about the center of gravity, the turning moment M_L of the aerodynamic force. By analogy

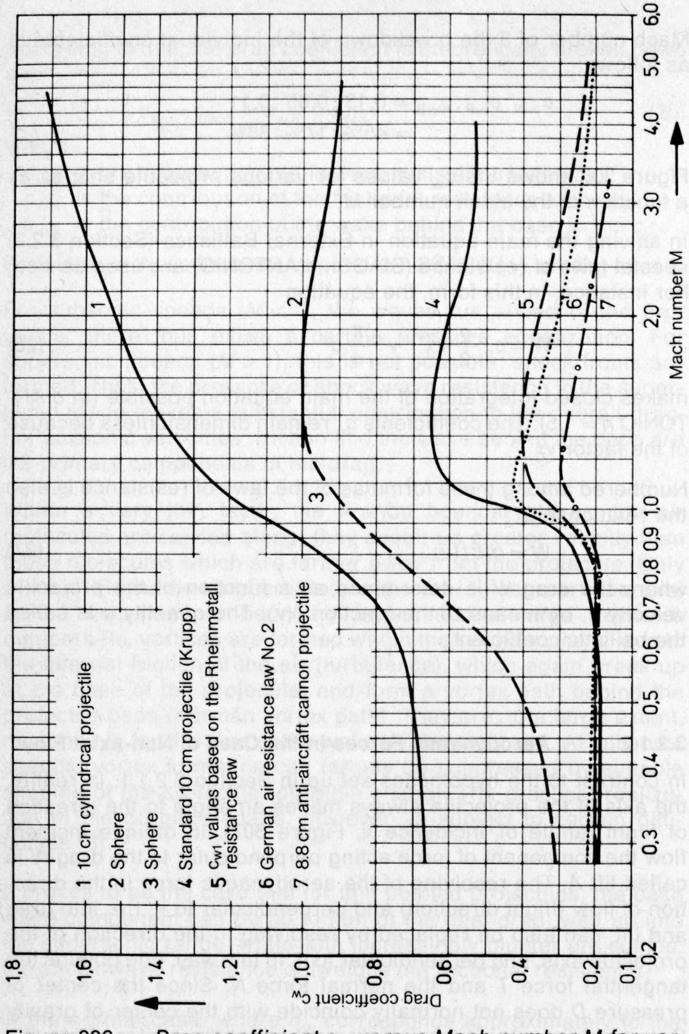

Figure 306. *Drag coefficient c_w versus Mach number M for various projectiles.*

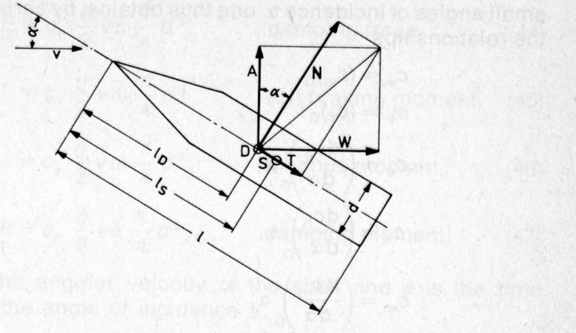

Figure 307. *Aerodynamic forces acting on the projectile.* v = incident flow velocity, α = incident flow angle (angle of incidence), D = center of pressure, S = center of gravity, d = projectile diameter, l = projectile length, l_S = the spacing between the center of gravity and the projectile tip, l_D = the spacing of the center of pressure from the projectile tip.

with the Prandtl expression (Equation (21)), using the corresponding coefficients $c_{(\ldots)}$, one can then write:

$$W = c_w \frac{\varrho}{2} v^2 \frac{\pi}{4} d^2, \quad \text{drag,} \tag{21}$$

$$A = c_a \frac{\varrho}{2} v^2 \frac{\pi}{4} d^2, \quad \text{lift,} \tag{28}$$

$$T = c_t \frac{\varrho}{2} v^2 \frac{\pi}{4} d^2, \quad \text{tangential force,} \tag{29}$$

$$N = c_n \frac{\varrho}{2} v^2 \frac{\pi}{4} d^2, \quad \text{normal force,} \tag{30}$$

$$M_L = c_m \frac{\varrho}{2} v^2 \frac{\pi}{4} d^3, \quad \text{turning moment of the aerodynamic force.} \tag{31}$$

For reasons of symmetry, c_w and c_t must be even functions; c_a, c_n and c_m must be uneven functions of the angle of incidence α. For