3. HYDRODYNAMIC CHARACTERISTICS OF PROPELLERS

The performance characteristics of a propeller can be divided into two groups; open water and behind hull properties.

a) Open Water Characteristics

The forces and moments produced by the propeller are expressed in terms of a series of non-dimensional characteristics. These non-dimensional terms expressing the general performance characteristics are:

Thrust coefficient	$K_T = \dfrac{T}{\rho n^2 D^4}$
Torque coefficient	$K_Q = \dfrac{Q}{\rho n^2 D^5}$
Advance coefficient	$J = \dfrac{V_A}{nD}$
Cavitation number	$\sigma = \dfrac{P_0 - P_V}{\frac{1}{2}\rho V^2}$

In order to establish those non-dimensional parameters, dimensional analysis can be applied to geometrically similar propellers.

Thrust, T and Torque, Q can be represented by the following functions depending upon the physical quantities involved:

$$T \approx f_1(\rho, D, V, g, n, P, \mu)$$
$$Q \approx f_2(\rho, D, V, g, n, P, \mu)$$

where

Quantities	**Symbols**	**Dimensions**
Thrust	T	ML/T^2
Torque	Q	ML^2/T^2
Diameter	D	L
Speed	V	L/T
Rate of rotation	n	1/T
Mass density of water	ρ	M/L^3
Viscosity of water	μ	M/LT
Acceleration due to gravity	g	L/T^2
Total static pressure	P	M/LT^2

L, T and M are the three basic quantities of mechanics, i.e. length, time and mass respectively.

Consider thrust equation as:

$$T = f(\rho^a D^b V^c g^d n^e P^f \mu^g)$$

and inserting the appropriate dimensions, it follows:

$$\frac{ML}{T^2} = (\frac{M}{L^3})^a L^b (\frac{L}{T})^c (\frac{L}{T^2})^d (\frac{1}{T})^e (\frac{M}{LT^2})^f (\frac{M}{LT})^g \tag{1}$$

Equating terms:

$$\begin{aligned} M:&\quad 1 = a + f + g && a = 1\text{-}f\text{-}g \\ L:&\quad 1 = -3a + b + c + d - f - g && b = 1 + 3a - c - d + f + g \\ T:&\quad -2 = -c - 2d - e - 2f - g && c = 2 - 2d - e - 2f - g \end{aligned} \tag{2}$$

By substituting a and c in b, it then follows:

$$b = 2 + d + e - g \tag{3}$$

By substituting (2) and (3) in (1)

$$\frac{ML}{T^2} = (\frac{M}{L^3})^{1-f-g} (L)^{2+d+e-g} (\frac{L}{T})^{2-2d-e-2f-g} (\frac{L}{T^2})^d (\frac{1}{T})^e (\frac{M}{LT^2})^f (\frac{M}{LT})^g \tag{4}$$

By rearranging the above terms:

$$\begin{aligned} \frac{ML}{T^2} = \frac{M}{L^3} L^2 (\frac{L}{T})^2 f \left[L(\frac{L}{T})^{-2} (\frac{L}{T^2}) \right]^d \left[L(\frac{L}{T})^{-1} (\frac{1}{T}) \right]^e \left[(\frac{M}{L^3})^{-1} (\frac{L}{T})^{-2} (\frac{M}{LT^2}) \right]^f \\ \left[(\frac{M}{L^3})^{-1} L^{-1} (\frac{L}{T})^{-1} (\frac{M}{LT}) \right]^g \end{aligned} \tag{5}$$

and inserting appropriate quantities

$$\begin{aligned} T = \rho D^2 V^2 f \left\{ (\frac{gD}{V^2})^d (\frac{nD}{V})^e (\frac{P}{\rho V^2})^f (\frac{\nu}{VD})^g \right\} \\ where\ \nu = \frac{\mu}{\rho} \text{ is the kinematic viscosity} \end{aligned} \tag{6}$$

$$C_T = \frac{T}{\frac{1}{2}\rho D^2 V^2} = f\left\{ \frac{gD}{V^2}, \frac{nD}{V}, \frac{P}{\rho V^2}, \frac{\nu}{VD} \right\} \tag{7}$$

where C_T is defined as thrust coefficient.

By following the above procedure similarly for the torque:

$$C_Q = \frac{Q}{\frac{1}{2}\rho D^3 V^2} = f\left\{\frac{gD}{V^2}, \frac{nD}{V}, \frac{P}{\rho V^2}, \frac{\nu}{VD}\right\} \tag{8}$$

where C_Q is defined as torque coefficient

From equations (7) and (8) it is clear that in order to achieve a flow similarity between two geometrically similar propellers (i.e., to achieve the same C_T and C_Q between a model propeller and actual propeller), the four non-dimensional parameters, $\frac{gD}{V^2}, \frac{nD}{V}, \frac{P}{\rho V^2}, \frac{\nu}{VD}$ should be the same for the two propellers. However this requirement may not be satisfied for all the test cases.

C_T and C_Q become infinitive when speed V approaches to zero. To avoid this undesirable situation V is replaced by nD term which does not make any difference in terms of dimensionality. Hence we have new thrust and torque coefficients defined as:

$$\begin{aligned} C_T \text{ becomes } K_T &= \frac{T}{\rho n^2 D^4} \\ C_Q \text{becomes } K_Q &= \frac{Q}{\rho n^2 D^5} \end{aligned} \tag{9}$$

On the other hand the four non-dimensional coefficients are:

$$\begin{aligned} Fn &= \frac{V^2}{gD} && \text{propeller Froude number} \\ Rn &= \frac{VD}{\nu} && \text{propeller Reynolds number} \\ J &= \frac{V}{nD} && \text{propeller advance coefficient} \\ \sigma &= \frac{P}{\rho V^2} && \text{propeller cavitation number} \end{aligned} \tag{10}$$

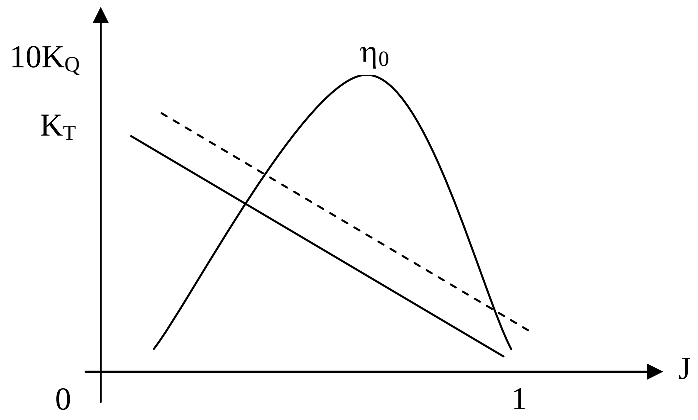
In addition to the thrust and torque coefficients, another open water characteristic is the open water efficiency η_0 defined as:

$$\eta_0 = \frac{P_T}{P_D} \tag{11}$$

where P_T is the thrust power while P_D is the delivered power and they are defined as $P_T = TV$ and $P_D = 2\pi Qn$

$$\eta_0 = \frac{P_T}{P_D} = \frac{TV}{2\pi Qn} = \frac{K_T \rho n^2 D^4 V}{2\pi K_Q n^2 D^5 n} = \frac{K_T}{2\pi}\frac{V}{nD}\frac{1}{K_Q} = \frac{K_T J}{2\pi K_Q} = \frac{J}{2\pi}\frac{K_T}{K_Q} \qquad (12)$$

These non-dimensional parameters are used to display open water diagrams (performance) of a propeller which gives characteristics of the powering performance of a propeller.



b) Propeller Hull Interaction – Wake

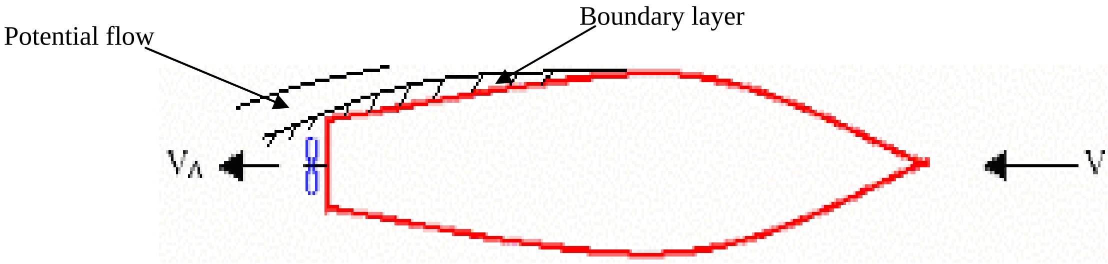
When a propeller operates behind the hull of a ship its hydrodynamic characteristics (i.e. thrust, torque and efficiency) differ from the characteristics of the same propeller operating in open water condition. This is mainly due to different flow conditions.

Theoretically the interaction phenomenon is caused by 3 main effects:

1. Wake gain
2. Thrust deduction
3. Relative-rotative efficiency

Wake gain:

The flow field around a propeller close to the hull is affected by the presence of the hull both because of the potential (non-viscous) nature and viscous nature (boundary layer growth) of the flow.



As a result, average speed of water through the propeller plane, V_A, is different usually less than the speed of the hull, V. The difference between the hull speed and the V_A is called wake velocity ($V - V_A$).

The ratio of the wake velocity to the hull speed V is known as the wake fraction:

$$w_T = \frac{V - V_A}{V}$$ also known as Taylor wake fraction.

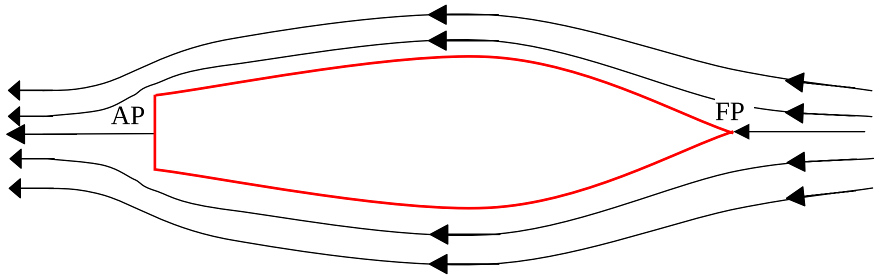
The other definition of the wake fraction was made by Frodue as:

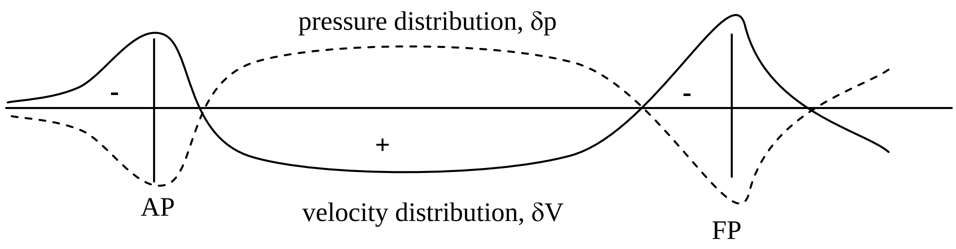
$$w_F = \frac{V - V_A}{V_A}$$

Wake gain or simply wake can be composed of three components:

Total wake = Potential wake + Viscous (frictional) wake + Wave-making wake

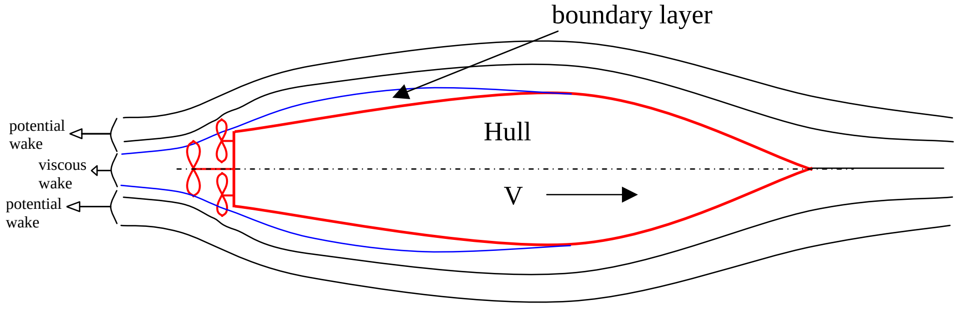
i- Potential or Displacement Wake Component:



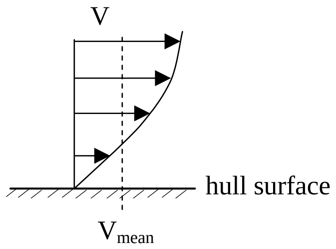


The potential flow past the hull causes an increased pressure around the stern where the streamlines are closing in. This means that, in this region, the relative velocity of the flow past the hull will be less than the speed of the hull and this will appear as a forward or positive wake increasing the wake speed.

ii- Frictional (Viscous) wake component



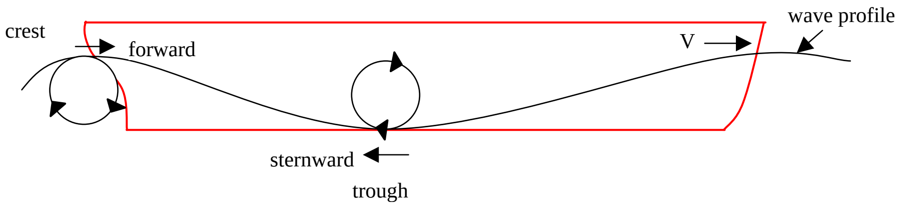
cross-section through the boundary layer:



Because of the viscous effects:

- Mean speed through the boundary layer (V_{mean}) less than the ship speed V.
- Frictional wake ≈ 80 to 90% of the total wake effects. Since single screw propeller mainly operates in a viscous (frictional) wake, the wake effect is so important.
- Twin screws work mainly in potential wake therefore the wake effect is relatively less important.

iii- Wave-making wake component:



The ship forms a wave pattern on the water and the water particles in the wave crest have a forward velocity due to their orbital motion, while in the troughs the velocity is sternward. This orbital velocity will give rise to a wake component which will be positive or negative depending upon the position of wave system in the vicinity of the propeller.

There is a crest (i.e. slow to medium speed ships) or trough (i.e. fast ships) of wave system at the propeller plane.

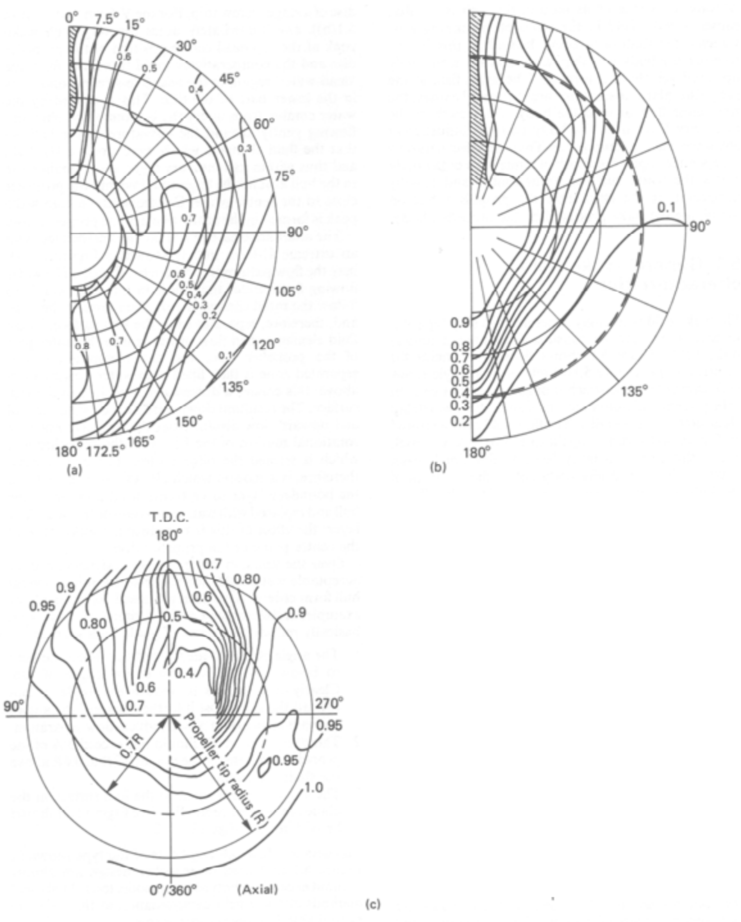
Wake definitions:

Nominal wake: The wake in the propeller plane without the propeller action or without the presence of the propeller is known as nominal wake.

Effective wake: When the effects of the propeller in nominal wake are taken into account one talks about the effective wake.

The following figures are typical values for w. They are based on model tests and not to be regarded as absolute due to the scale effects and other factors which are neglected.

w (wake fraction)		
$0.5C_B$-0.05	for single screw	Taylor's formulae
$0.55C_B$-0.20	for twin screw	
0.30	C_B=0.70	for moderate speed cargo ship
0.4 ≈ 0.5	C_B=0.80 ≈ 0.85	for large bulk carrier
0.25	C_B=0.60 ≈ 0.65	for containership
0.10 ≈ 0.15	C_B=0.50	for twin screw ferry
0.05	at cruising speed	for high speed frigate
-0.05	at full speed	



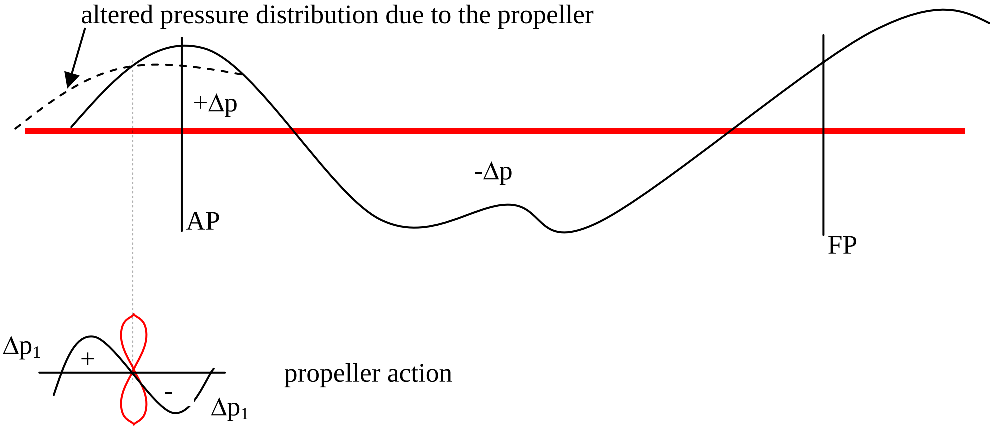
Typical wake contours: a) U form hull; b) V form hull; c) twin-screw hull

Thrust deduction:

Propeller accelerates the flow ahead of itself, thereby:

- increasing the rate of shear in the boundary layer and hence increasing the frictional resistance of the hull
- reducing pressure over the rear of the hull and hence increasing the pressure resistance

Because of the above reasons, the action of the propeller is to alter the resistance of the hull (usually to increase it) by an amount which is approximately proportional to the thrust. This means that the thrust (T) developed by the propeller must exceed the towed resistance of the hull (R).



Augment of resistance (increase) ΔR:

$$\Delta R = (\Delta p - \Delta p_1) ds$$

where ds is the hull surface element.

By defining t as “thrust deduction factor”:

$$t = \frac{\Delta R}{T} = \frac{T - R}{T}$$

$$R = T(1 - t)$$

The thrust deduction can be estimated by using semi-empirical formulae. A common practice is to measure it at model scale using a stock propeller with an approximate diameter and with the required loading at the design speed.

The thrust deduction depends on streamlining and propeller clearances relative to the hull and rudder. It also increases with fullness.

Typical values of t are given below:

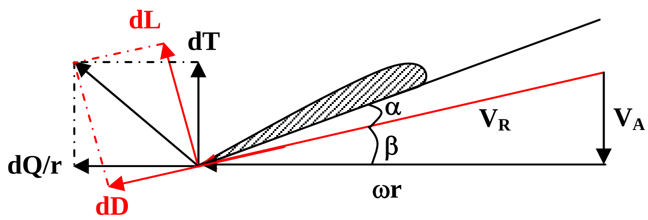
t (thrust deduction factor)		
0.6w	for single screw	Taylor's formulae
w	for twin screws	
0.25w+0.14	for twin screws with bossings	
0.7w+0.06	for twin screws with brackets	
$0.3C_B$	for modern single screws	

Relative-rotative efficiency:

The efficiency of a propeller in the wake behind a hull is not the same as the propeller operating in open water. This is because:

i. Level of turbulence in the flow is very low in an open water condition whilst it is very high in the wake behind the hull.
ii. The flow behind a hull is very non-uniform so that flow conditions at each radius at the propeller plane are different from the conditions in open water case.

High turbulence level affects the lift and drag on the propeller blades and hence its efficiency. Therefore a propeller is deliberately designed for the radial variation in wake (wake adapted propellers) to achieve a further gain.



The relative-rotative efficiency η_R is defined as the ratio of the power delivered to a propeller producing the same thrust in open water (P_{D0}) and in behind (P_D) conditions such that:

$$\eta_R = \frac{P_{D0}}{P_D}$$

where P_{D0}: Delivered power in open water condition
P_D : Delivered power in behind condition

or

$$\eta_R = \frac{P_{D0}}{P_D} = \frac{\eta_B}{\eta_0} = \frac{\text{Efficiency behind hull}}{\text{Efficiency in open water}}$$

η_R is usually $\eta_R \approx 0.96$ to 1.04 depending upon the propeller type.

Propulsive efficiency and propulsion factors:

In power prediction, w, t and η_R are frequently referred as propulsion factors or propulsion coefficients.

The relationship between the propulsive efficiency η_D (or Quasi propulsive coefficient, QPC) can be established as follows:

$$\eta_D = \frac{P_E}{P_D}$$

$$= \frac{P_E}{P_T}\frac{P_T}{P_{D0}}\frac{P_{D0}}{P_D} = \frac{RV}{TV_A}\eta_0\eta_R$$

$$= \frac{T(1-t)V}{TV(1-w)}\eta_0\eta_R$$

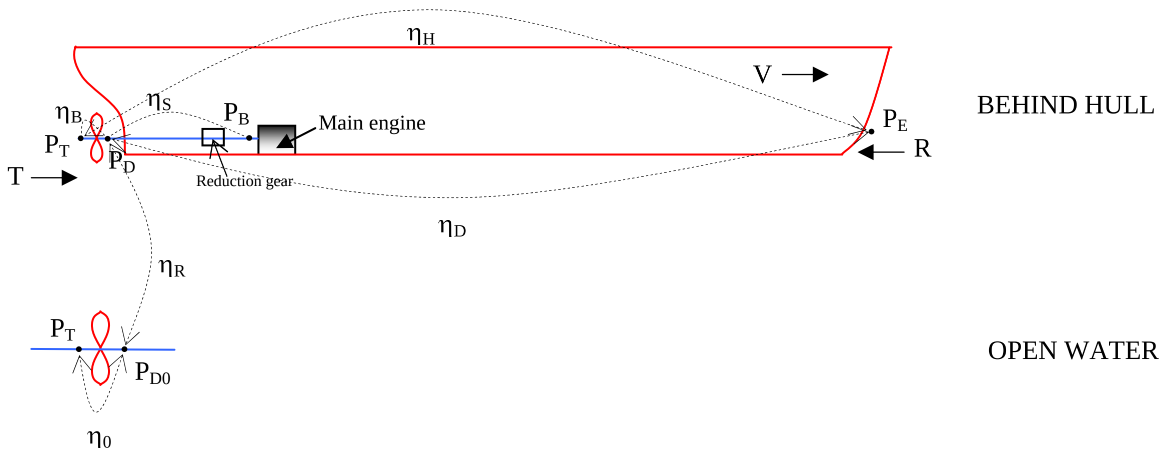
$$\eta_D = \frac{1-t}{1-w}\eta_0\eta_R$$

and $\frac{1-t}{1-w} = \eta_H$ as hull efficiency by definition. Typical values for hull efficiency

1.0 ≈ 1.25 for single screws
0.98 ≈ 1.05 for twin screws

$$\eta_D = \eta_H\eta_0\eta_R$$

Summary of efficiencies in powering:



$$\eta_D = \frac{P_E}{P_D}$$

$$\eta_H = \frac{P_E}{P_T}$$

$$\eta_B = \frac{P_T}{P_D}$$

$$\eta_R = \frac{P_{D0}}{P_D}$$

$$\eta_0 = \frac{P_T}{P_{D0}}$$

$$\eta_S = \frac{P_D}{P_B}$$

$$\eta_B = \eta_0 \eta_R \qquad \eta_D = \eta_0 \eta_R \eta_H$$

T	Thrust
R	Resistance
V	Ship speed
P_T	Thrust power
P_D	Delivered power in behind hull condition
P_{D0}	Delivered power in open water condition
P_B	Brake power
P_E	Effective power
η_0	Open water efficiency
η_R	Relative-rotative efficiency
η_B	Behind hull efficiency
η_S	Shaft transmission efficiency
η_H	Hull efficiency
η_D	Propulsive efficiency

c) Standard Series Propeller Data

Systematic open water tests with series of model propellers were performed to form a basis for propeller design. The series were generated from a parent form such that certain parameters influencing the performance of the propeller were varied systematically. These parameters are:

Diameter, D Pitch, P	usually D fixed, P/D varied
Blade Area Ratio, BAR Number of blades, Z	BAR & Z varied
Blade shape Blade thickness	kept constant

There are several series developed over the years. These are Wageningen B Series (or Troost Series), AU Series, Gawn Series, Gawn-Burril (KCA) Series, Ma Series, Schaffran Series. We will be dealing with the most acceptable two series, Wageningen B and Gawn Series.

i- Wageningen B propeller series:

Amongst the series, one of the most extensive and widely used for fixed pitch, merchant ship (slow to medium speed) model propeller series is the WAGENINGEN OR TROOST B SERIES.

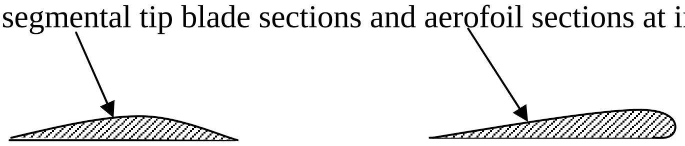
The basic form of B-series is simple. They have modern sections and have good performance characteristics. About 210 propellers were tested in Wageningen (today known as MARIN) model tank in the Netherlands.

The family of models of fixed diameter was generated by varying:

P/D	0.5 to 1.4
Z	2 to 7
A_E/A_0	0.3 to 1.05

The basic characteristics of B-Series are such that they have:

- 250 mm diameter and r_h/R is 0.167 (r_h is the hub radius)
- constant radial pitch distribution at outer radii R
- small skew
- 15° backward rake angle with linear rake distribution
- a blade contour with fairly wide tips
- segmental tip blade sections and aerofoil sections at inner radii



- no consideration of cavitation

Each B-Series is designated by **BZ.y**

where B represents series type (B)
Z represents the number of blades (2 to 7)
y represents BAR=A_E/A_0 (0.3 to 1.05)

For example B-4.85

ii- Gawn series

This series of propellers comprised a set of 37 three-bladed propellers covering a range of pitch ratios and BAR:

P/D	0.4 to 2.0
BAR	0.2 to 1.1

The entire series were tested in the No:2 towing tank at Admiralty Experimental Works (AEW) Haslar, UK and presented by Gawn.

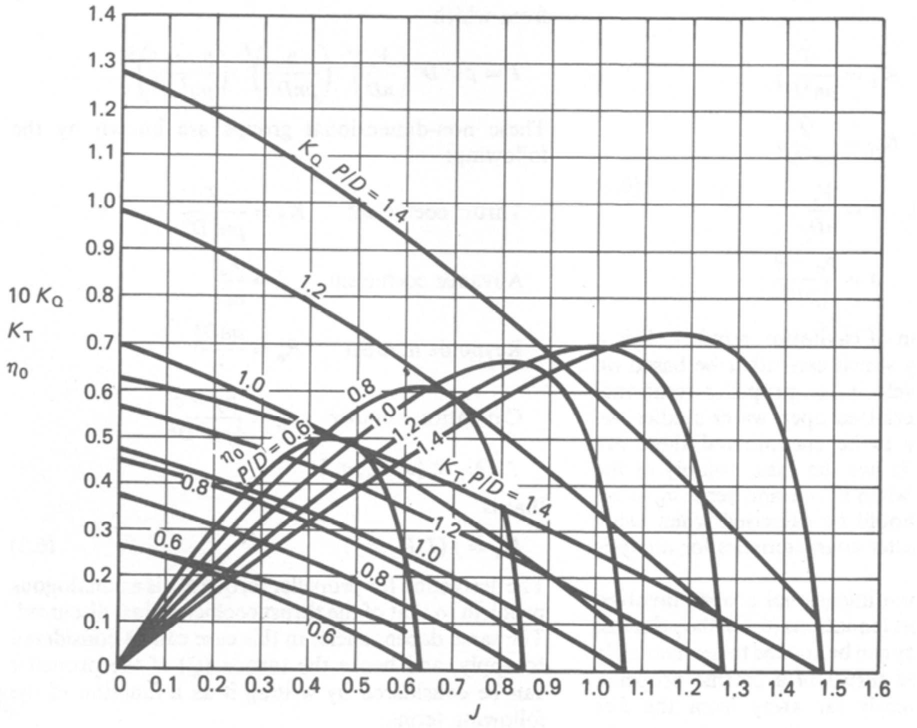
These series have:

- a diameter of 508 mm (20 inches)
- segmental blade sections
- constant blade thickness ratio s_i/D=0.06
- a hub diameter of 0.20D
- no cavitation characteristics given

iii- Representation of Series

The representation of systematic open water diagrams may differ depending on the design options. The most widely used diagrams are K_T-K_Q-J diagrams B_p-B_u-δ diagrams and μ-σ-ϕ diagrams.

K_T-K_Q-J diagrams:



$$K_T = \frac{T}{\rho n^2 D^4} \qquad K_Q = \frac{Q}{\rho n^2 D^5}$$

In addition to the above coefficients Taylor presented the following constants with the following units:

Power coefficient

$$B_p = \frac{N P_D^{1/2}}{V_a^{2.5}}$$

N (rpm)
P_D (HP)
V_a (knots)

or

$$B_p = 1.158 \frac{N P_D^{1/2}}{V_a^{2.5}}$$

N (rpm)
P_D (kW)
V_a (knots)

or

$$B_p = 33.46 \sqrt{\frac{K_Q}{J^5}}$$

SI units
N (rps)
P_D (W)
V_a (m/s)

Thrust coefficient

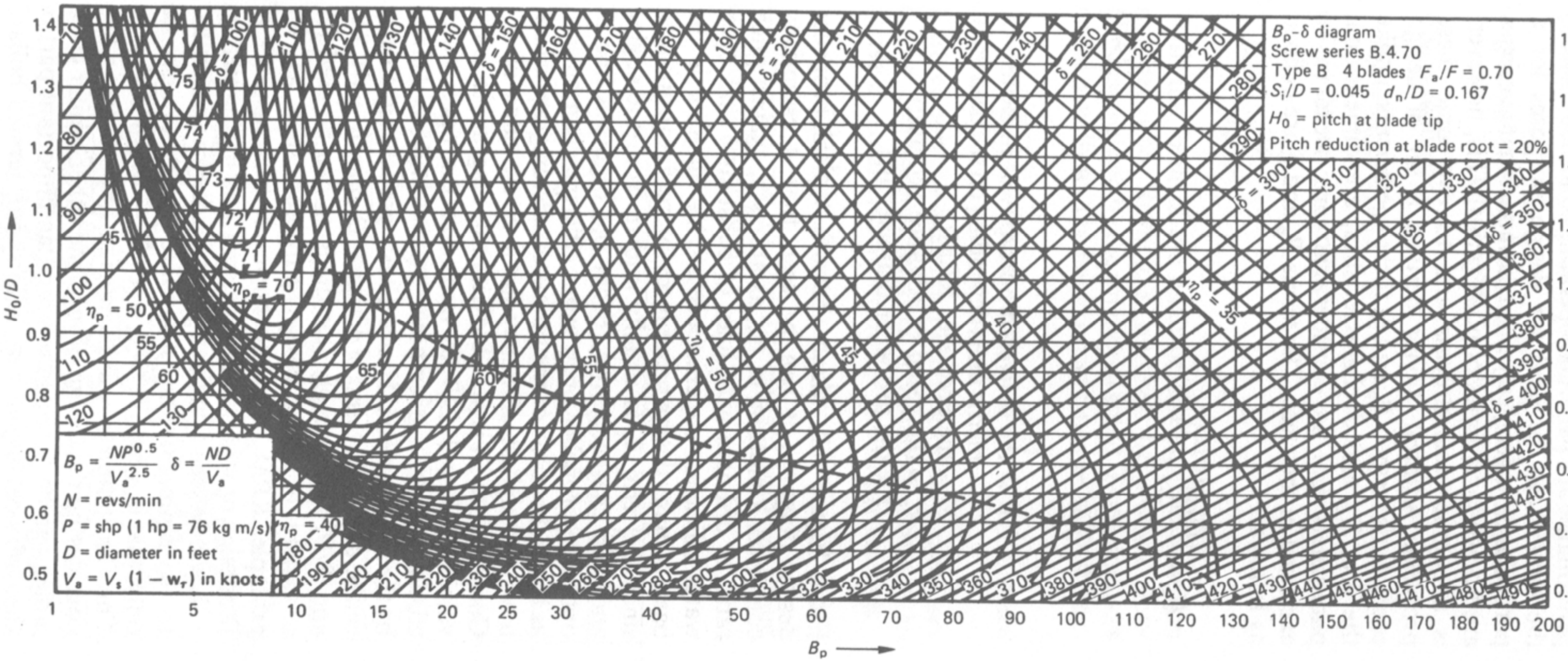
$$B_u = \frac{N U^{1/2}}{V_a^{2.5}}$$

N (rpm)
U=P_T (HP)
V_a (knots)

or

	$B_u = 1.158 \frac{NU^{1/2}}{V_a^{2.5}}$	N (rpm) P_D (kW) V_a (knots)
	or	
	$B_u = 13.35 \sqrt{\frac{K_T}{J^4}}$	SI units N (rps) P_D (W) V_a (m/s)
Advance constant	$\delta = \frac{ND}{V_a}$	N (rpm) D (feet) V_a (knots)
	or	
	$\delta = \frac{101.23}{J}$	SI units N (rps) D (m) V_a (m/s)
Propeller efficiency	$\eta_0 = \frac{P_T}{P_D}$	

B_p-δ and B_u-δ diagrams were obtained from K_T-K_Q-J diagrams.



B_p–δ diagram
Screw series B.4.70
Type B 4 blades $F_a/F = 0.70$
$S_i/D = 0.045$ $d_n/D = 0.167$
H_0 = pitch at blade tip
Pitch reduction at blade root = 20%
$B_p = \frac{NP^{0.5}}{V_a^{2.5}}$ $\delta = \frac{ND}{V_a}$
N = revs/min
P = shp (1 hp = 76 kg m/s)
D = diameter in feet
$V_a = V_s(1 - w_T)$ in knots
H_0/D
B_p
$\eta_p = 70$
$\eta_p = 50$
$\eta_p = 40$
$\eta_p = 35$
$\delta = 100$
$\delta = 150$
$\delta = 200$
$\delta = 250$
$\delta = 300$
$\delta = 350$
$\delta = 400$