$$W_g = \int_0^L \left\{\psi(x)\left[E.A(x)\frac{d^2U}{dx^2} + p\right]\right\} dx = 0$$

$$W_g = \int_0^L \left\{\psi(x).E.A(x)\frac{d^2U}{dx^2} + \psi(x).p\right\} dx = 0$$

IPP pour calculer l'intégrale faible W_f :

$$W_g = \int_0^L \left\{\psi(x).E.A(x)\frac{d^2U}{dx^2} + \psi(x).p\right\} dx = 0$$

Je me concentre sur D (en rouge):

$$D = \int_0^L \left\{\psi(x).E.A(x)\frac{d^2U}{dx^2}\right\} dx$$

$$\int u.v.dw = [u.v.w] - \int u.w.dv - \int v.w.du$$

$$D = \int_0^L \left\{\psi(x).E.A(x)\frac{d^2U}{dx^2}\right\} dx = \left[\psi(x).A(x).\frac{dU}{dx}\right] - \int_0^L \psi(x).\frac{dA(x)}{dx}.\frac{dU}{dx}dx - \int_0^L \frac{d\psi(x)}{dx}.A(x).\frac{dU}{dx}dx$$

$\frac{dU}{dx} = 0$ donc $\left[\psi(x).A(x).\frac{dU}{dx}\right] = 0$

Donc W_f :

$$W_f = -\int_0^L \left[\psi(x).E.\frac{dA(x)}{dx}.\frac{dU}{dx}\right] dx - \int_0^L \left[\frac{d\psi(x)}{dx}.E.A(x).\frac{dU}{dx}\right] dx + \int_0^L [\psi(x).p]dx = 0$$

$$\int_0^L \left[\psi(x).E.\frac{dA(x)}{dx}.\frac{dU}{dx}\right] dx + \int_0^L \left[\frac{d\psi(x)}{dx}.E.A(x).\frac{dU}{dx}\right] dx = \int_0^L [\psi(x).p]dx \quad \textbf{(1)}$$

On a 2 nœuds : $U_x(x) = N_1(x).U_x^{\langle 1\rangle} + N_2(x).U_x^{\langle 2\rangle}$

Méthode de Galerkin :

$$\psi_1(x) = N_1(x).\delta U_x^{\langle 1\rangle} \rightarrow \frac{d\psi_1(x)}{dx} = \left(\delta U_x^{\langle 1\rangle}\right).\frac{dN_1(x)}{dx}$$

$$\psi_2(x) = N_2(x).\delta U_x^{\langle 2\rangle} \rightarrow \frac{d\psi_2(x)}{dx} = \left(\delta U_x^{\langle 2\rangle}\right).\frac{dN_2(x)}{dx}$$

(1) Devient :

- Au nœud 1 (i=1) :

$$\int_0^L \left[\left(N_1(x).\delta U_x^{\langle 1\rangle}\right).E.\frac{dA_1}{dx}.\left\{\frac{dN_1(x)}{dx}U_x^{\langle 1\rangle} + \frac{dN_2(x)}{dx}U_x^{\langle 2\rangle}\right\}\right] dx + \int_0^L \left[\left(\delta U_x^{\langle 1\rangle}\right).\frac{dN_1(x)}{dx}.E.A_1.\left\{\frac{dN_1(x)}{dx}U_x^{\langle 1\rangle} + \frac{dN_2(x)}{dx}U_x^{\langle 2\rangle}\right\}\right] dx = p\int_0^L [N_1(x).\delta U_x^{\langle 1\rangle}]dx$$

=0

=0

- Au nœud 2 (i=2)

$$\int_0^L \left[\left(N_2(x).\delta U_x^{\langle 2\rangle}\right).E.\frac{dA_2}{dx}.\left\{\frac{dN_1(x)}{dx}U_x^{\langle 1\rangle} + \frac{dN_2(x)}{dx}U_x^{\langle 2\rangle}\right\}\right] dx + \int_0^L \left[\left(\delta U_x^{\langle 2\rangle}\right).\frac{dN_2(x)}{dx}.E.A_2.\left\{\frac{dN_1(x)}{dx}U_x^{\langle 1\rangle} + \frac{dN_2(x)}{dx}U_x^{\langle 2\rangle}\right\}\right] dx = p\int_0^L [N_2(x).\delta U_x^{\langle 2\rangle}]dx$$

=0 =0

Ecriture sous forme de matrice :

$$E\int_0^L \begin{bmatrix} \frac{dN_1(x)}{dx}.A_1.\frac{dN_1(x)}{dx} & \frac{dN_1(x)}{dx}.A_1.\frac{dN_2(x)}{dx} \\ \frac{dN_2(x)}{dx}.A_2.\frac{dN_1(x)}{dx} & \frac{dN_2(x)}{dx}.A_2.\frac{dN_2(x)}{dx} \end{bmatrix} dx.\begin{bmatrix} U_x^{\langle 1\rangle} \\ U_x^{\langle 2\rangle} \end{bmatrix} = \int_0^L \begin{bmatrix} p.N_1 \\ p.N_2 \end{bmatrix} dx$$

$$N_1(x) = \frac{X-L}{0-L} = 1 - \frac{X}{L} \qquad \frac{dN_1(x)}{dx} = -\frac{1}{L}$$

$$N_2(x) = \frac{X-0}{L-0} = \frac{X}{L} \qquad \frac{dN_2(x)}{dx} = \frac{1}{L}$$

$$E\int_0^L \begin{bmatrix} A_1.\frac{1}{L^2} & -A_1.\frac{1}{L^2} \\ -A_2.\frac{1}{L^2} & A_2.\frac{1}{L^2} \end{bmatrix} dx.\begin{bmatrix} U_x^{\langle 1\rangle} \\ U_x^{\langle 2\rangle} \end{bmatrix} = \int_0^L \begin{bmatrix} p.\left(1-\frac{X}{L}\right) \\ p.\frac{1}{L} \end{bmatrix} dx$$

$$E\begin{bmatrix} \frac{A_1}{L} & -\frac{A_1}{L} \\ -\frac{A_2}{L} & \frac{A_2}{L} \end{bmatrix} \begin{bmatrix} U_x^{\langle 1\rangle} \\ U_x^{\langle 2\rangle} \end{bmatrix} = \begin{bmatrix} \frac{p.L^2}{2} \\ \frac{p.L^2}{2} \end{bmatrix}$$

$$\underbrace{\frac{E}{L}\begin{bmatrix} A_1 & -A_1 \\ -A_2 & A_2 \end{bmatrix}}_{K^e} \underbrace{\begin{bmatrix} U_x^{\langle 1\rangle} \\ U_x^{\langle 2\rangle} \end{bmatrix}}_{u^e} = \underbrace{\begin{bmatrix} \frac{P.L^2}{2} \\ \frac{P.L^2}{2} \end{bmatrix}}_{f^e}$$

$$\boxed{K^e \; . \; u^e \; = \; f^e}$$

Dernière question :

Je reprends et je modifie le résultat précédemment trouvé :

=0

$$\frac{E}{L}\begin{bmatrix} A_1 & -A_1 \\ -A_2 & A_2 \end{bmatrix} \begin{bmatrix} U_x^{\langle 1\rangle} \\ U_x^{\langle 2\rangle} \end{bmatrix} = \begin{bmatrix} 0 \\ p \end{bmatrix}$$

$$\frac{E}{L}.A_2.U_x^{\langle 2\rangle} = p$$

$$A_2 = 2.A$$

D'où : $U_x^{\langle 2\rangle} = \frac{P.L}{2.E.A}$