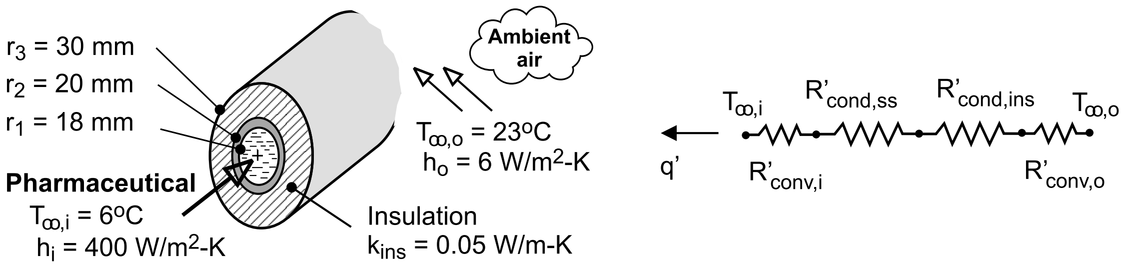
## PROBLEM 3.39

**KNOWN:** Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

**FIND:** (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

**PROPERTIES:** *Table A-1*, St. St. 304 (~280K): $k_{st} = 14.4$ W/m·K.

**ANALYSIS:** (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_i h_i} + \frac{\ln(r_2/r_i)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi(0.018\text{m})400\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(20/18)}{2\pi(14.4\ \text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.020\text{m})6\ \text{W/m}^2\cdot\text{K}}$$

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.33\right)\text{m}\cdot\text{K/W} = 1.35\ \text{m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23-6)^\circ\text{C}}{1.35\,\text{m}\cdot\text{K/W}} = 12.6\ \text{W/m} \qquad <$$

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins} + R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(30/20)}{2\pi(0.05\ \text{W/m}\cdot\text{K})} = 1.29\ \text{m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi(0.03\text{m})6\ \text{W/m}^2\cdot\text{K}} = 0.88\ \text{m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.29 + 0.88\right)\text{m}\cdot\text{K/W} = 2.20\,\text{m}\cdot\text{K/W}$$

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and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{17°C}{2.20 \text{ m} \cdot \text{K/W}} = 7.7 \text{ W/m}$$

**COMMENTS:** (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i}$ = 279K, large surroundings at $T_{sur} = T_{\infty,o}$ = 296K, and an emissivity of $\varepsilon = 0.7$, the heat gain due to net radiation exchange with the surroundings is $q'_{rad} = \varepsilon\sigma(2\pi r_2)\left(T_{sur}^4 - T_s^4\right) = 7.7 \text{ W/m}$. Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of $h_o$ with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

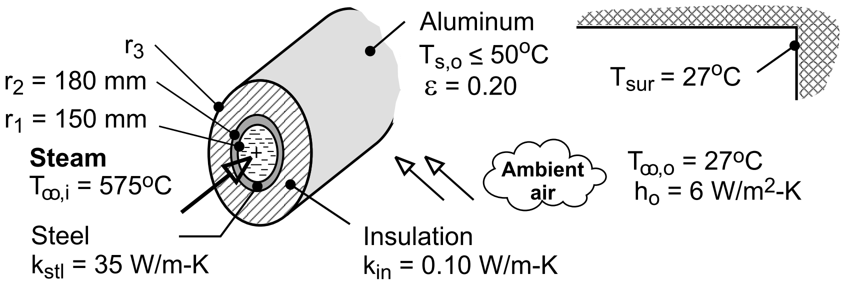
(3) The critical radius is $r_{cr} = k_{ins}/h \approx 8$ mm $< r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.

## PROBLEM 3.40

**KNOWN:** Diameter, wall thickness and thermal conductivity of steel tubes. Temperature of steam flowing through the tubes. Thermal conductivity of insulation and emissivity of aluminum sheath. Temperature of ambient air and surroundings. Convection coefficient at outer surface and maximum allowable surface temperature.

**FIND:** (a) Minimum required insulation thickness (r3 – r2) and corresponding heat loss per unit length, (b) Effect of insulation thickness on outer surface temperature and heat loss.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) One-dimensional radial conduction, (3) Negligible contact resistances at the material interfaces, (4) Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$), (5) Negligible conduction resistance for aluminum sheath, (6) Constant properties, (7) Large surroundings.

**ANALYSIS:** (a) To determine the insulation thickness, an energy balance must be performed at the outer surface, where $q' = q'_{conv,o} + q'_{rad}$. With $q'_{conv,o} = 2\pi r_3 h_o \left(T_{s,o} - T_{\infty,o}\right)$, $q'_{rad} = 2\pi r_3 \;\varepsilon\sigma \left(T_{s,o}^4 - T_{sur}^4\right)$, $q' = \left(T_{s,i} - T_{s,o}\right) / \left(R'_{cond,st} + R'_{cond,ins}\right)$, $R'_{cond,st} = \ell n\left(r_2/r_1\right)/2\pi k_{st}$, and $R'_{cond,ins} = \ell n\left(r_3/r_2\right)/2\pi k_{ins}$, it follows that

$$\frac{2\pi\left(T_{s,i} - T_{s,o}\right)}{\dfrac{\ell n\left(r_2/r_1\right)}{k_{st}} + \dfrac{\ell n\left(r_3/r_2\right)}{k_{ins}}} = 2\pi r_3\left[h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{sur}^4\right)\right]$$

$$\frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\ \text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(r_3/0.18\right)}{0.10\ \text{W/m}\cdot\text{K}}} = 2\pi r_3\left[6\ \text{W/m}^2\cdot\text{K}\left(323 - 300\right)\text{K} + 0.20\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 300^4\right)\text{K}^4\right]$$

A trial-and-error solution yields $r_3$ = 0.394 m = 394 mm, in which case the insulation thickness is

$$t_{ins} = r_3 - r_2 = 214\,\text{mm}$$ <

The heat rate is then

$$q' = \frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\,\text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(0.394/0.18\right)}{0.10\,\text{W/m}\cdot\text{K}}} = 420\,\text{W/m}$$ <

(b) The effects of $r_3$ on $T_{s,o}$ and $q'$ have been computed and are shown below.

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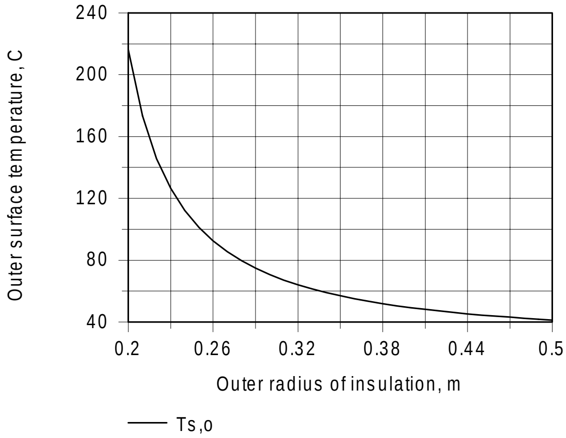
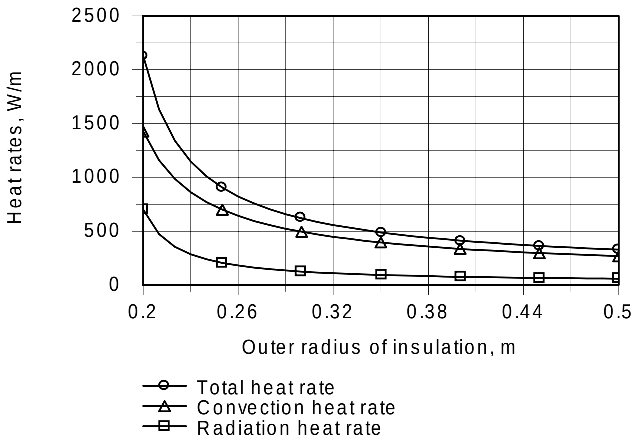
**PROBLEM 3.40 (Cont.)**




Beyond $r_3 \approx 0.40$m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

**COMMENTS:** Note that the thermal resistance of the insulation is much larger than that for the tube wall. For the conditions of Part (a), the radiation coefficient is $h_r = 1.37$ W/m, and the heat loss by radiation is less than 25% of that due to natural convection $\left(q'_{rad} = 78\,W/m,\ q'_{conv,o} = 342\ W/m\right)$.