and $$m\frac{d^2y}{dt^2} = -mg \tag{2}$$

with the initial conditions ($t=0$)

$$x=0, \tag{3}$$

$$y=0, \tag{4}$$

$$\frac{dx}{dt} = v_0 \cos\vartheta_0, \tag{5}$$

and $$\frac{dy}{dt} = v_0 \sin\vartheta_0. \tag{6}$$

where
- x is the horizontal coordinate of the trajectory,
- y is the vertical coordinate of the trajectory,
- t is the flight time to a point on the trajectory (x, y),
- ϑ_0 is the angle of departure,
- m is the mass of the projectile.

By integrating Equations (1) and (2), and using the initial conditions (3) and (4), one obtains the coordinates of any point on the trajectory at a time t:

$$x = v_0 t \cos\vartheta_0 \tag{7}$$

and $$y = v_0 t \sin\vartheta_0 - \frac{g}{2}t^2, \tag{8}$$

from which, after eliminating the time t, the equation for the trajectory becomes:

$$y = x\tan\vartheta_0 - \frac{g x^2}{2v_0^2\cos^2\vartheta_0}. \tag{9}$$

This is a parabola which is symmetrical about the ordinates y_G of the apex.

Figure 301 shows a trajectory parabola, indicating the most important quantities.

The relationships between the quantities in Table 301 can be obtained from Equations (7) to (9), (12) and (13).

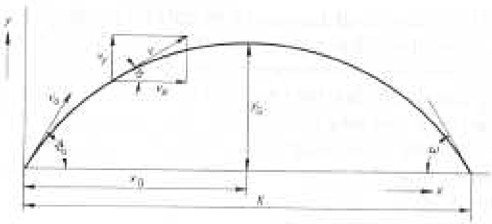

Figure 301. *Trajectory parabola.*

Of these equations, only the relationship between the vertex height y_G and the total flight time T is mentioned here because of its practical significance:

$$y_G = \frac{g}{8}T^2. \tag{10}$$

This expression, known in the literature as Haupt's equation, is only exact in a vacuum, but can also be used in the atmosphere as a good approximation for estimating the vertex height [1].

3.1.2 Parabola of Safety

If, for a given initial velocity v_0, the angle of departure ϑ_0 is varied, one then obtains a family of parabolic trajectories with the enveloping curve symmetrical about the y axis (Figure 302):

$$y = \frac{v_0^2}{2g} - \frac{g x^2}{2v_0^2}. \tag{11}$$

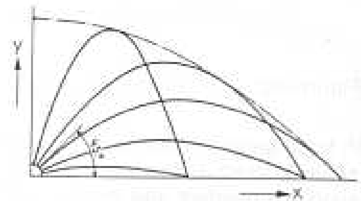

Figure 302. *Parabola of safety.*

Outside of this so-called parabola of safety, no target can be hit with the v_0 specified. Each point *on* the enveloping curve can be reached by only *one* trajectory. However, all points *inside* the enveloping parabola can be reached by two trajectories. Targets lying below the trajectory for $\vartheta_0 = 45°$ can be reached both by low angle fire ($\vartheta_0 < 45°$) and high angle fire ($\vartheta_0 > 45°$). For targets on the horizontal line the sum of departure angles ϑ_0 of these two trajectories is 90°.

3.1.3 Firing on an Inclined Plane; Lifting the Trajectory

If the line to the target makes an angle γ with the horizontal line, we have for the range

$$X_\gamma = \frac{2v_0^2}{g} \frac{\cos\vartheta_0 \sin(\vartheta_0 - \gamma)}{\cos^2\gamma}, \tag{12}$$

and for the flight time

$$T = \frac{2v_0}{g} \frac{\sin(\vartheta_0 - \gamma)}{\cos\gamma}. \tag{13}$$

In order to hit the target *A* (Figure 303), the trajectory must be lifted, i.e., the superelevation angle ϑ_1 must be added to the angle of site γ. One terms the overall angle ϑ_0 the elevation.

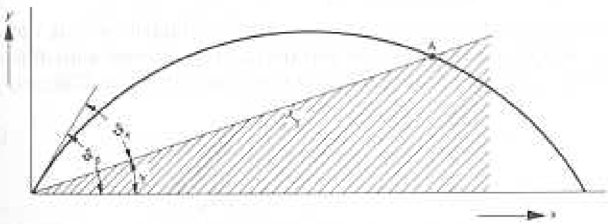

Figure 303. *Firing on an inclined plane.*

If, for sloping ground (uphill or downhill), the same superelevation angle is used as for level ground, then a round will normally fall short for positive site angles, and overshoot for negative ones. However, for small site angles γ, this error is small.