

$$\begin{aligned}
\frac{\partial [F_{\alpha\beta} F^{\alpha\beta}]}{\partial(\partial_\mu A_\nu)} &= F_{\alpha\beta} \frac{\partial F^{\alpha\beta}}{\partial(\partial_\mu A_\nu)} + \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} F^{\alpha\beta} = 2 \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} F^{\alpha\beta} \\
&= 2 [\delta_\alpha^\mu \delta_\beta^\nu - \delta_\beta^\mu \delta_\alpha^\nu] F^{\alpha\beta} = 2 [F^{\mu\nu} - F^{\nu\mu}] = 4F^{\mu\nu}.
\end{aligned}$$