$$\frac{\partial\left[F_{\alpha\beta}F^{\alpha\beta}\right]}{\partial(\partial_\mu A_\nu)} = F_{\alpha\beta}\frac{\partial F^{\alpha\beta}}{\partial(\partial_\mu A_\nu)} + \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)}F^{\alpha\beta} = 2\frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)}F^{\alpha\beta}$$
$$= 2\left[\delta_\alpha{}^\mu\delta_\beta{}^\nu - \delta_\beta{}^\mu\delta_\alpha{}^\nu\right]F^{\alpha\beta} = 2\left[F^{\mu\nu} - F^{\nu\mu}\right] = 4F^{\mu\nu}.$$